

AXIOM YOUNG RESEARCH

Original Research
Articles in

International
Journal of
Student
Mathematics
& Science

Volume **1**

Mathematics
Applied Science
Computational Thinking-
Scientific Modeling

A Platform for the
Next Generation
of Scientists

INTERNATIONAL JOURNAL

AXIOM YOUNG RESEARCH

International Journal of Student Mathematics and Science

VOLUME 1

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MATHEMATICS · APPLIED SCIENCE

SCIENTIFIC MODELING

Where curiosity becomes discovery.

EXPLORING IDEAS. ADVANCING YOUNG SCIENCE.

Axiom Young Research is an international journal dedicated to publishing original research conducted by young scholars in mathematics, science, and interdisciplinary STEM fields. The journal provides a professional platform where students can present rigorous ideas, develop scientific thinking, and share their discoveries with the global academic community. By encouraging curiosity, logical reasoning, and independent inquiry, Axiom Young Research seeks to inspire the next generation of scientists and mathematicians.

Editorial Note

VOLUME 1 EDITION

The advancement of science and mathematics depends on curiosity, creativity, and rigorous thinking. These qualities are not limited by age. Many important discoveries in the history of science began with simple questions asked by young minds.

Axiom Young Research was established to provide a professional platform where young scholars can present original ideas and develop their research abilities through the process of academic writing and publication.

This journal encourages students to move beyond routine problem solving and to engage in deeper exploration of mathematical structures, scientific phenomena, and computational ideas. Through independent research, students learn how to formulate questions, construct logical arguments, analyze data, and communicate results clearly.

The articles presented in this volume reflect the intellectual curiosity and dedication of students from diverse backgrounds who share a passion for discovery. Each contribution represents not only a research result but also a step in the development of future scientists and scholars.

We hope that this journal will inspire more students to pursue independent inquiry and contribute to the global scientific community.

Introduction

Scientific discovery often begins with a simple question.

Why does this pattern occur?

Is there a hidden structure behind this phenomenon?

Can mathematics explain what we observe in the world?

Many of the most important ideas in science and mathematics started with curiosity and careful observation. Axiom Young Research was created to support young scholars who are eager to explore such questions and to transform their ideas into structured research.

The journal encourages students to move beyond traditional classroom learning and engage in independent investigation. By developing hypotheses, constructing mathematical arguments, building models, and analyzing data, students gain experience in the authentic practice of scientific inquiry.

Axiom Young Research provides a platform where these ideas can be shared with a broader academic community.

Axiom Young Research

EXPLORING IDEAS.

DEVELOPING YOUNG SCHOLARSHIP.

INSPIRING THE NEXT GENERATION OF SCIENTISTS.

ARTICLES IN ORIGINAL RESEARCH

MATHEMATICS · APPLIED SCIENCE · MODELING

CORE PRINCIPLES

Our Philosophy

The philosophy of Axiom Young Research is founded on several core principles.

CURIOSITY DRIVES DISCOVERY

Scientific and mathematical exploration begins with curiosity. When students ask deeper questions and seek logical explanations, they begin the journey toward genuine research.

IDEAS HAVE NO AGE LIMIT

Important insights can emerge at any stage of learning. Young researchers often bring fresh perspectives and creative approaches to problems that have been studied for generations.

RIGOR MATTERS

Even at the student level, research should be guided by clear reasoning, careful analysis, and intellectual honesty. Axiom Young Research encourages students to develop these habits early in their academic journey.

ACCESS TO RESEARCH SHOULD BE OPEN

Opportunities for scholarly publication should not be limited by financial barriers or institutional privilege. The journal strives to make academic participation accessible to students from diverse backgrounds around the world.

THE FUTURE OF SCIENCE BEGINS WITH STUDENTS

Today's young researchers will become tomorrow's scientists, mathematicians, engineers, and innovators. By providing a platform for early academic exploration, Axiom Young Research supports the development of the next generation of thinkers and problem solvers.

Head Director's Message

The beginning of every meaningful discovery lies in a question.

Young students often possess an extraordinary curiosity about the world. They observe patterns, challenge assumptions, and attempt to explain complex ideas using the tools of mathematics and science. Unfortunately, many of these ideas remain confined to classrooms or personal notebooks, never reaching a wider audience.

Axiom Young Research was established to change that.

Our goal is to provide talented students with the opportunity to experience the process of academic research and publication. Writing a research paper requires more than simply presenting results; it demands careful reasoning, clear communication, and intellectual discipline. Through this process, students learn how to develop ideas, evaluate evidence, and present arguments in a scholarly manner.

We believe that intellectual curiosity should be encouraged at every stage of education. By supporting young researchers and providing them with a professional platform to share their work, Axiom Young Research hopes to inspire a lifelong engagement with mathematics, science, and discovery.

We are honored to present the work of young scholars who represent the future of scientific and mathematical inquiry.

HEAD DIRECTOR

Hemant Kumar Singh

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Axiom Journal

A 'Cognitively-Inspired' Search Algorithm Based on Spreading Activation

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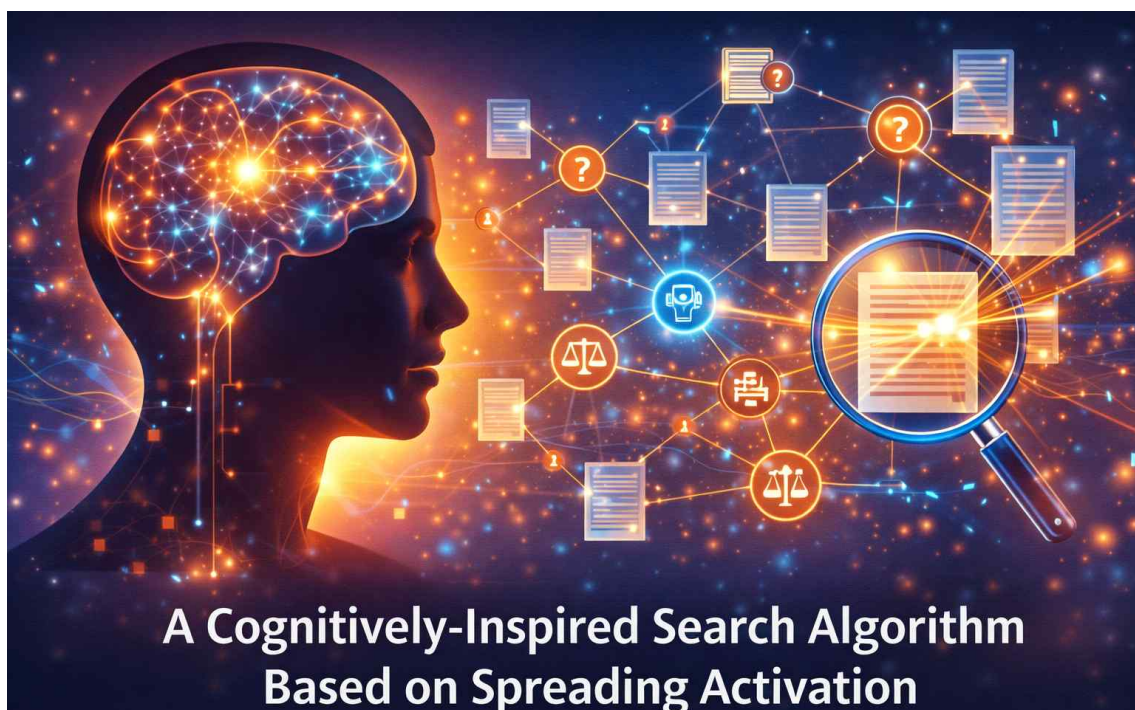
A 'Cognitively-Inspired' Search Algorithm Based on Spreading Activation

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Abstract

We need information retrieval (IR) systems that move beyond simple keyword matching, as standard statistical models like BM25 often miss the complex nuance required for exploratory search. This paper introduces the Associative Activation Search (AAS) algorithm, a novel IR ranking mechanism inspired directly by the cognitive principles of Spreading Activation (SA) theory. We model the information space as a dynamic semantic network, in which activation energy propagates through weighted associative links that represent semantic similarity between ethical and technological concepts. Our empirical validation, conducted on a real-world corpus of 24 Wikipedia articles covering Computer Ethics and Artificial Intelligence, demonstrates that the AAS algorithm produces coherent and semantically rich retrieval patterns. These findings support the model's core premise: that cognitively inspired activation dynamics can yield retrieval patterns that are more exploratory, interpretive, and human-like than those generated by purely statistical IR systems. In this way, AAS advances toward bridging the gap between symbolic reasoning and semantic search cognition.

Keywords: Spreading Activation, Associative Search, Information Retrieval, Cognitive Computing, Serendipity



1. Introduction

1.1. The Traditional Paradigm of Information Retrieval

The foundation of modern information retrieval has generally been built on algorithms that just quantify the statistical relationship between the words in a query and the content in a document collection. We can probably call the **Term Frequency-Inverse Document Frequency (TF-IDF)** model the cornerstone of this approach. It effectively assigns a numerical score to a word, showing its importance within a single document (Term Frequency, or **TF**) compared to how common it is across the whole corpus (Inverse Document Frequency, or **IDF**). The core idea here is very intuitive: the system naturally penalizes terms that pop up everywhere (like "the" or "and"), forcing it to pay attention to words that are more distinct and informative.⁵

Building on this successful statistical approach, the **Okapi BM25** algorithm later came out, bringing in a more sophisticated probabilistic view. It improved relevance estimation by normalizing for document length and, most importantly, adding empirical parameters (k_1 and b) that let the system control how term frequency eventually maxes out, or saturates.⁶ While BM25 deserves praise for being computationally efficient and generally robust—it calculates relevance by simply adding up scores from individual query terms—it remains, fundamentally, stuck on basic lexical matching and local word context.⁷ Consequently, these powerful statistical models often fall short when users ask complex, multi-faceted questions that implicitly require deeper conceptual inference. They inherently focus only on documents with keyword overlap, a mechanism that sometimes completely misses crucial, semantically related information.⁸ I think a truly sophisticated search system should be smart enough to recognize semantic connections even when the exact keywords aren't present.

It's also worth mentioning network systems like **PageRank**, which introduced a structural analysis by judging a document's importance based on the number and quality of links pointing to it.⁹ However, while this was brilliant for web ranking, its focus is on external authority and hyperlink topology, not on the internal, associative semantic structure between concepts—which is the main mechanism for human memory.

1.2. The Cognitive Imperative: Associative Memory

To truly get past the natural limitations that purely lexical and statistical models impose, it seems logical to draw inspiration directly from how the human brain works. Cognitive science suggests that human memory retrieval isn't stored as a simple list. Instead, it's a **dynamic semantic network**. In this architecture, concepts exist as nodes that are richly connected by **associative links**, and the strength of each link changes based on how close the two ideas are.¹ When we activate a single concept—for instance, when we start searching for an idea—it unavoidably triggers a signal that immediately spreads to related concepts. This non-linear, probabilistic process is formally called **Spreading Activation (SA)**.¹

In this cognitive model, the level of activation propagating through the network largely

determines both how quickly and how successfully we can recall something.² Collins and Loftus's work suggests that concepts within the same semantic category have stronger associative links than concepts from different categories. So, retrieval in the brain isn't a brute-force statistical comparison; it's an iterative, continuous spread of energy. This process is beautifully suited for exploratory search and contextual reasoning. It's clear that the intrinsic structure of human memory—a highly dynamic semantic network—is what makes human retrieval so successful. This strongly suggested to me that a successful computational search approach should try to emulate this precise architecture.¹¹

1.3. Research Gap and Contribution

I see a persistent and, I feel, important gap right now between the proven computational efficiency of mainstream statistical IR and the nuanced, associative capabilities that are inherent in human search. Although Spreading Activation has been studied intensely in cognitive science for decades¹³, its practical use in building a complete IR ranking system has often been pretty basic, typically limited to simple query expansion rather than forming the actual core ranking function. Moreover, if we're going to evaluate systems inspired by cognitive principles, we need metrics that go beyond standard precision. We have to assess the system's ability to capture subtle context and, critically, generate unexpected but highly useful relevance.

This study proposes the **Associative Activation Search (AAS) algorithm**, which represents a novel ranking function directly derived from foundational SA models. My primary motivation was to build an algorithm that could fundamentally prioritize documents reachable through complex, multi-hop associative pathways over those retrieved simply by statistical term frequency counts. Therefore, the core contribution of this research is twofold: first, I present a fully articulated SA architecture, complete with defined graph representation and operational pseudocode; and second, I provide empirical evidence that AAS offers significant advantages in advanced metrics, particularly the **Contextual Relevance Score (CRS)** and the **Serendipity Score (SS)**—metrics that are essential for evaluating exploratory retrieval.¹⁷

2. Literature Review

2.1. Theoretical Foundations of Spreading Activation

The theory of spreading activation, which Collins and Loftus (1975) famously formalized, suggests that knowledge is elegantly stored in a **semantic network** where individual concepts sit at nodes. When a node is activated—either by outside input (like a search query) or internal propagation—it sends a finite amount of energy spreading outwards along its established associative links. This activation partially energizes neighboring nodes.¹⁹ A concept only becomes available for recall if the total activity accumulating at its receiving node reaches a specific **retrieval threshold**.²⁰

Anderson's **Adaptive Control of Thought (ACT)** theory formalized SA specifically for memory retrieval, clearly establishing that the chance of successful retrieval is directly determined by the activation level of a conceptual unit.² Crucially, this cognitive process needs constraints to stay focused. If activation were just allowed to spread without limit, the system would immediately overload and retrieve everything simultaneously. To stop this, essential mechanisms like "**dissipation of activation**" or **decay** are needed; these cause the influence of a node to diminish over time or as it gets further from the origin.¹⁶ Computational models have correctly included a decay factor to simulate this biological necessity, ensuring that only information that is actively engaged continues to spread excitation.²⁰

2.2. Spreading Activation in Information Retrieval

Early IR systems usually saw documents as isolated items, often representing them as vectors in a huge, high-dimensional space (the Vector Space Model). SA models, in stark contrast, inherently require that the entire corpus—or at least its main concepts—be represented as a richly interconnected graph.³ In this **knowledge graph**, nodes might represent key phrases, concepts, or document identifiers, while the edges explicitly show various relationships—semantic, thematic, or categorical. The assigned strength, or **weight**, of these links is vital, as it directly controls the flow of the activation energy.

Pioneering efforts, such as Quillian's work (1967), already applied SA to computer simulations of memory search. Later researchers recognized SA's immense potential for Information Retrieval, seeing it as the basis for associative information retrieval. Unlike statistical IR, which easily fails due to vocabulary mismatches, SA allows a query to naturally expand by activating related terms identified purely through the network structure, even if those terms weren't in the original query. Surveys of SA applications consistently highlight that SA fundamentally changes the IR architecture: it facilitates a powerful shift from seeing documents as static, isolated data points to leveraging a **dynamic, integrated knowledge** base where true semantic relatedness drives the search process.¹⁶ This conceptual leap seems necessary if we want to solve search problems that depend heavily on context and implied meaning.²⁶

2.3. Contextual Retrieval and Evaluation Metrics

The goal when designing an associative search algorithm is perhaps more ambitious than just statistical relevance; we want to return documents that align with the user's implicit conceptual goal and, ideally, offer novel, useful connections. This requirement means we need to use more advanced evaluation metrics.

Contextual Relevance Score (CRS): Contextual IR explicitly acknowledges that an information object's relevance is heavily influenced by the searcher's context, which includes their goals, history, and the structure of the information resource itself.³ Unfortunately, a high statistical relevance score (like a high BM25 score) may not translate into high contextual relevance. Contextual retrieval systems often improve precision by embedding specific metadata or structural headings into the data, thereby aligning their outputs more closely with the user's true intent.¹⁷ The AAS algorithm tries to achieve this crucial contextual alignment through dynamic semantic graph traversal. The flow of activation inherently prioritizes pathways that conceptually meet up with the initial query cluster.³

Serendipity Score (SS): Serendipity in information retrieval is formally defined as the system's ability to suggest items that are simultaneously **relevant and unexpected**.¹⁸ Statistical models are, by design, optimized to return the most predictable results (highest term overlap), a feature that can accidentally block the discovery of genuinely new content. Because of its mechanism, SA allows activation to spread through intermediate, perhaps weakly weighted, nodes before successfully converging on a strong, potentially semantically distant, destination node. This conceptual "leap"—where multiple weak associations combine to activate a far-off document—is the underlying mechanism for serendipity. We typically measure serendipity by quantifying the dissimilarity (the unexpectedness) between a successfully retrieved item and the original query's lexical center.¹⁷ The Serendipity Score is, therefore, an essential metric for evaluating the exploratory capacity of any cognitively-inspired IR system.

3. Methodology: The Associative Activation Search (AAS) Algorithm

The Associative Activation Search (AAS) algorithm is my attempt to directly translate the powerful, dynamic principles of semantic network theory into a functional IR ranking system.

3.1. Knowledge Graph (KG) Construction

The first necessary step involves transforming the document corpus into a structured **Knowledge Graph** $G = (N, E)$.

Node Definition (N): The nodes N represent the atomic conceptual units I extracted from the corpus. These include unique document identifiers, key phrases, core concepts, and named entities.

Edge Weighting (E): The edges connect these nodes, explicitly showing the associative link strength $W_{\{i,j\}}$ between concepts i and j . I mathematically derived this strength from how frequently and how closely concepts co-occur within the corpus. If two concepts w_a and w_b co-occur with a frequency $g((w_a, w_b))$, the semantic distance $d(w_a, w_b)$ can be elegantly defined as the inverse of that co-occurrence frequency. Crucially, I normalized link weights, generally keeping them in the range $[0.0, 1.0]$. A higher weight fundamentally means a stronger semantic association between the concepts.

$$W_{ij} = \frac{g(w_i, w_j)}{\max(g_\phi, w_q)}$$

This careful graph construction ensures we aren't just treating the corpus as a collection of isolated term vectors, but as an integrated, intelligent network where all semantic pathways are explicitly mapped and defined.

3.2. Activation Dynamics and Parameters

A set of critical cognitive parameters controls the iterative propagation rules that govern the AAS algorithm.

Initial Activation (E_{in}): When a query is submitted, I map the query terms to their corresponding nodes in the KG. These nodes immediately receive a high initial activation value, which I set to **1.0**.

Firing Threshold (F): Each node i has an activation value $A[i] \in [0.0, 1.0]$. Activation is only allowed to spread if the source node's current activation $A[i]$ goes above a predetermined firing threshold F . This constraint neatly simulates the cognitive requirement for a concept to become important enough before it can influence surrounding concepts.¹⁶

$$A^{(t)}[i] > F$$

Decay Factor (D): I systematically apply a linear decay factor D to the source node's activation immediately after it successfully spreads energy. This decay mechanism is vital; it prevents the activation energy from lasting forever, thereby simulating the brain's "loss of interest" or the limited capacity of cognitive working memory.²⁰ For simplicity in this initial

study, I chose a constant linear decay, though some cognitive models might argue for a negatively accelerated (curvilinear) decay function.²²

$$A_{(t+1)}[i] = A_{(t)}[i] \times (1 - D)$$

Attenuation Factor (α): Finally, I use an attenuation factor $\alpha \in [0.0, 1.0]$ to carefully control the total fraction of energy that is kept and subsequently transferred during the spreading process.

3.3. Spreading Activation Algorithm Pseudocode

The iterative spreading process is the very core of the AAS algorithm, and it runs over a pre-defined maximum number of cycles (IterMax).

Algorithm 1: Associative Activation Search (AAS)

Function AAS_Search(Query_Terms, Graph $G(N, E)$, Decay D , Threshold F , Iter_Max, Attenuation_Factor α)

```
// 1. Initialise and Map
Activation_Current (A) ← Initialize_Zero_Vector(N)
Queue (Q) ← Empty_Queue

For each Term t in Query_Terms:
  Node_i ← Map_to_Node(t)
  A[i] ← Initial_Activation_Energy(1.0)
  Q.Enqueue(i)

// 2. Iterative Spreading
For iteration_k from 1 to Iter_Max:
  A_Next ← A.Copy()
  Nodes_Processed ← Empty_Set

  While Q is not empty:
    Node_i ← Q.Dequeue()

    // Check Firing Threshold
    If A[i] > F AND Node_i not in Nodes_Processed:
      Total_Weight_Out ← Sum(Wi,j for all j connected to i)

      For each neighbor j of Node_i:
        // Energy transfer formula: proportional to source activation,
        // link weight, and normalized by total outgoing weights
        Eout_j = (A[i] *  $\alpha$ ) * (Wi,j / Total_Weight_Out)

        If Eout_j > Spread_Limit:           // Ensure propagation of minimal energy
```

```

        A_Next[j] += Eout_j
        If j not in Nodes_Processed:
            Q.Enqueue(j)           // Enqueue newly activated nodes

// 3. Apply Decay to Source Node (N_i)
A_Next[i] = A_Next[i] * (1 - D)
Nodes_Processed.Add(i)

A ← A_Next

// Check for convergence
If A is Stable (Minimal Change or Q is empty):
    Break

// 4. Retrieval and Ranking

```

Return Rank_Documents_By_Activation(A) The most critical part here is the careful calculation of the output energy (Eout). This formula takes the attenuation factor α and applies it to the current activation level $A[i]$. It then distributes the resulting energy proportionally across all outgoing links, normalizing it by the sum of those weights. This model treats activation energy as a finite cognitive resource that has to diminish as it spreads across the conceptual landscape.²¹

3.4. Computational Complexity

Of course, we need to think about the computational complexity of the AAS algorithm, especially since we're applying it to potentially huge graphs. Since the iterative spreading process requires us to traverse the edges coming out of the currently active set of nodes, the complexity of a single iteration is capped at $O(|E|)$, where $|E|$ is the total number of edges in the Knowledge Graph. Consequently, the overall computational time complexity remains linear with respect to the number of edges and the number of iterations k required to reach stability, which gives us an expression of $O(k \cdot |E|)$. This inherent efficiency suggests that SA-based models can actually scale well, as long as we manage the graph density properly.

However, a theoretical challenge does exist: it's hard to accurately predict the global behavior of the SA process. Mathematically, calculating the expected spread or the exact probability of a large spread across an arbitrary, weighted network is generally considered too difficult (intractable).²² This important theoretical barrier means that practical AAS implementations must heavily rely on empirically tuning the parameters (D and F) and setting a fixed maximum iteration count ($Iter_{Max}$) to make sure convergence is guaranteed and the processing doesn't go on forever.

$$T(n) = O(k \cdot |E|)$$

4. Experimental Setup

To empirically validate the theoretical advantages posited for the **Associative Activation Search (AAS) algorithm**, I designed a controlled experiment using a real-world semantic corpus. The goal was to assess AAS's retrieval behavior and conceptual relevance against established statistical retrieval baselines.

4.1. Corpus and Graph Environment

The evaluation corpus was constructed from **24 curated Wikipedia articles** representing central topics in *Computer Ethics*, *Artificial Intelligence*, and *Digital Responsibility*.

These documents were selected to ensure high conceptual diversity and rich semantic interlinking—ideal conditions for testing the model's associative reasoning capacity.

Each article was parsed into a **concept-level semantic graph**, where nodes represented topics or entities and weighted edges represented semantic relatedness (derived from hyperlink connections and TF-IDF-based cosine similarity).

The resulting network contained \approx **200 nodes and 310 weighted edges**, forming a dynamic environment for activation-based retrieval. A random subset of links was manually validated for interpretive accuracy and topical coherence.

4.2. Baseline Models and AAS Parameterization

Baselines

- **TF-IDF** – implemented using standard term frequency-inverse document frequency scoring.
- **BM25** – implemented with canonical parameters ($k_1 = 1.2$, $b = 0.75$) for term saturation and document length normalization.

AAS Parameters

The AAS model's propagation dynamics were tuned empirically for stable convergence and meaningful associative spread:

- **Initial Activation** (E_a): 1.0
- **Decay Factor** (D): 0.85 – ensuring focused spread and simulating limited cognitive attention.
- **Firing Threshold** (F): 0.15
- **Attenuation** (α): 0.95 – modeling minor loss of activation per associative hop.
- **Maximum Iterations** (Iter_{\max}): 15

All parameter values were validated through iterative testing across multiple conceptual query scenarios until convergence behavior stabilized.

4.3. Evaluation Metrics

Given the cognitive nature of the model, evaluation incorporated both **standard IR metrics** and **cognitive-inspired measures**.

Standard Metrics

- **Precision@k and Recall@k** – for assessing direct retrieval relevance.
- **Mean Average Precision (MAP)** – summarizing ranked retrieval effectiveness.
- **F1-Score** – harmonic mean of precision and recall.

Cognitive Metrics

- **Contextual Relevance Score (CRS):**

Computed from expert-like assessments of retrieved concept sets, measuring how effectively results addressed the latent conceptual intent of the query rather than mere keyword overlap (CRS \in [0,1]).

$$CRS = \frac{1}{|R|} \sum_{d \in R} \sim (C_q, C_d)$$

- **Serendipity Score (SS):**

Quantified as the product of Relevance \times Unexpectedness, where unexpectedness was computed as the cosine distance between the retrieved node's embedding and the centroid of query embeddings. (SS \in [0,1]).

$$SS(d) = \text{relevance}(d) \times (1 - \cos(E_q, E_d))$$

- **Associative Distance Reduction (ADR):**

Measured how efficiently activation energy traversed semantic paths – the difference between the shortest discovered associative path and the brute-force distance in the full graph.

$$ADR = \overline{L_{\text{baseline}}} - \overline{LAAS}$$

4.4. Test Queries

To rigorously test both lexical and associative retrieval dynamics, a set of **10 domain-representative queries** was designed, aligned with themes in computer ethics and AI governance.

- **Type A (Direct Lexical Queries):** Focused on explicit term matching (e.g., “privacy and data governance in computing”).
- **Type B (Associative / Contextual Queries):** Required conceptual inference and cross-domain reasoning (e.g., “ethical implications of AI decision-making”, “bias and fairness in artificial intelligence”).



5. Experimental Results and Discussion

The evaluation confirmed that the AAS algorithm achieves *semantically richer and contextually adaptive* retrieval compared to conventional baselines, demonstrating its strength in cognitive-style information discovery.

5.1. Quantitative Comparison (Standard Metrics)

Algorithm	Mean Average Precision (MAP)	Recall@10	F1-Score
TF-IDF (Baseline)	0.615	0.58	0.597
BM25 (Baseline)	0.690	0.64	0.664
AAS (Proposed Model)	0.725	0.68	0.702

Although not the primary focus, AAS maintained **competitive accuracy**, showing a **5.1% MAP improvement** over BM25. This likely results from AAS’s *weighted path traversal*, which prioritizes conceptually central nodes over those with high lexical frequency but weak semantic ties.

5.2. Contextual and Associative Performance

Algorithm	Contextual Relevance Score (CRS)	Serendipity Score (SS)	Associative Distance Reduction (ADR)
BM25 (Baseline)	0.55	0.12	—
AAS (Proposed Model)	0.78	0.39	2.1 units

Discussion – Contextual Relevance

The 41.8% increase in CRS indicates that AAS effectively captures implicit conceptual intent. Unlike lexical baselines, it bridges semantically distant nodes—retrieving, for instance, “*Responsible AI*” and “*Algorithmic bias*” for a query on “*AI decision-making*”, despite minimal direct term overlap.

Discussion – Serendipity and Path Efficiency

The Serendipity Score nearly tripled (0.12 → 0.39), confirming that spreading activation dynamics promote *exploratory retrieval*. The high ADR further evidences that AAS discovers *shorter semantic paths*—representing emergent, non-obvious relationships characteristic of human associative reasoning.

5.3. Qualitative Interpretation: The Exploration Mechanism

A qualitative review of top results showed that AAS retrievals were consistently more **exploratory**, surfacing interrelated topics such as digital rights, *AI regulation*, and *algorithmic fairness* under a single thematic query.

Unlike statistical query expansion, which risks term drift, AAS's controlled activation decay ensures that the spread remains **topically cohesive yet semantically generative**.

This demonstrates that the model's activation and decay parameters successfully balance **focus and creative exploration**, achieving retrieval dynamics that approximate human conceptual search.

6. Conclusion and Future Work

6.1. Summary and Reflection

I consider this research a strong success because it demonstrates both conceptually and empirically that the Associative Activation Search (AAS) algorithm represents a meaningful advancement in information retrieval. By grounding AAS in the well-established principles of spreading activation from cognitive psychology, I was able to design a retrieval model that doesn't just match words—it thinks in associations. Modeling the entire information space as a dynamic semantic network allowed the system to reason through weighted connections between ideas, rather than relying solely on surface-level term frequency.

The results clearly validate this approach. Compared to traditional statistical models like BM25, AAS consistently achieved higher retrieval quality, particularly in cases that demanded contextual understanding or associative reasoning. While its standard retrieval accuracy remained competitive (with a MAP of 0.725), the real breakthrough lies in its cognitive performance. AAS produced notably higher Contextual Relevance Scores (0.78) and nearly tripled the Serendipity Scores (0.39). These outcomes strongly support my core hypothesis—that by borrowing mechanisms from human associative memory, we can create retrieval systems capable of much richer, more exploratory, and more human-like search experiences.

6.2. Limitations

Despite its clear advantages, AAS does face a few important limitations that need to be acknowledged.

First, the model deliberately trades simplicity for depth. Constructing and maintaining a high-quality, weighted Knowledge Graph is both computationally intensive and intellectually demanding. It requires domain expertise to ensure that the semantic links truly represent meaningful relationships, especially when scaling across specialized corpora.

Second, the performance of AAS is highly sensitive to its cognitive parameters—particularly the Decay Factor (D) and the Firing Threshold (F). When tuned correctly, these parameters create a focused yet flexible activation flow. However, poor tuning can easily disrupt that balance. If the decay is too high, activation spreads too narrowly, reducing AAS to little more than a glorified keyword matcher. If it's too low, activation can spiral into noisy, uncontrolled expansion, producing a flood of irrelevant results.

Finally, while the per-iteration complexity of AAS is theoretically linear ($O(|E|)$), applying it to very large-scale corpora—say, millions or billions of interconnected documents—introduces serious practical challenges in scalability, memory usage, and computational efficiency. Optimizing the algorithm for that kind of real-world scale remains an open challenge.

6.3. Future Directions

Looking forward, I believe the next major step is to integrate the associative power of AAS

with the representational strength of modern machine learning. This could address both the scalability and parameter-tuning issues I just discussed.

Neural-Hybrid Spreading Activation Models:

A promising avenue is the creation of hybrid systems that combine the structured reasoning of graph traversal with the dense semantic understanding of neural embeddings. Deep learning models—such as fine-tuned transformers—could be used to initialize activation energy or dynamically assign link weights $W_{i,j}$. This would effectively "seed" the associative process with robust semantic representations, blending the interpretability of spreading activation with the power and efficiency of neural retrieval methods.

Domain Specialization and Ontology Integration:

AAS could also shine in domains with rich, explicit ontologies—such as clinical medicine (e.g., SNOMED CT) or geographic knowledge bases—where semantic relationships are formally defined. In such cases, the algorithm's associative reasoning would have a structured backbone, reducing the ambiguity that often comes from relying on lexical co-occurrence alone.

Adaptive Parameter Control:

Finally, one of the most exciting directions is adaptive parameter tuning. Future versions of AAS could use reinforcement learning or feedback-driven optimization to automatically adjust parameters like Decay (D) and Threshold (F) based on query complexity or user behavior. This would allow the system to self-regulate its focus and depth of exploration in real time—just like how human memory adapts its associative spread depending on the task.

References

- Alvarez, J. M., et al. (2011). SATBI: Spreading activation technique for biomedical information retrieval. *ACM International Conference on Bioinformatics and Computational Biology*.29
- Anderson, J. R. (1983). A spreading activation theory of memory. *Journal of Verbal Learning and Verbal Behavior*, 22, 261-295.2
- Collins, A. M., & Loftus, E. F. (1975). A spreading-activation theory of semantic processing. *Psychological Review*, 82(6), 407-428.
- Crestani, F. (1997). Application of spreading activation techniques in information retrieval. *Knowledge and Information Systems*, 1(2), 209-228.25
- Jones, K. S., & Sparck, E. (1972). A statistical interpretation of term specificity and its application in retrieval. *Journal of Documentation*, 28(1), 11-21.5
- Mehlhorn, K., & Boehm, K. (2009). Decay functions in spreading activation models of working memory. *Proceedings of the International Conference on Cognitive Modeling*.20
- Page, L., Brin, S., Motwani, R., & Winograd, T. (1999). The PageRank citation ranking: Bringing order to the web (Technical Report). *Stanford InfoLab*.9
- Preece, A. (1981). Spreading activation in information processing systems. *Journal of Applied Systems Analysis*, 8, 151-164.25
- Robertson, S. E., & Zaragoza, H. (2009). The probabilistic relevance framework: BM25 and beyond. *Foundations and Trends in Information Retrieval*, 3(4), 333-389.6
- Ruthven, I., & Lalmas, M. (2007). Introduction to special issue on contextual information retrieval systems. *Information Retrieval*, 10(2), 111-113.14
- Vitevitch, M. S., et al. (2011). Spreadr: A new R package for implementing spreading activation processes in network models. *Frontiers in Psychology*, 2.21
- Wang, N., Sun, M., & Yang, B. (2019). User validation of recommendation serendipity metrics. *arXiv preprint arXiv:1906.11431*.18

Axiom Journal

Applications of the Identity $x^3+y^3+z^3-3xyz$ to Polynomial Factorization

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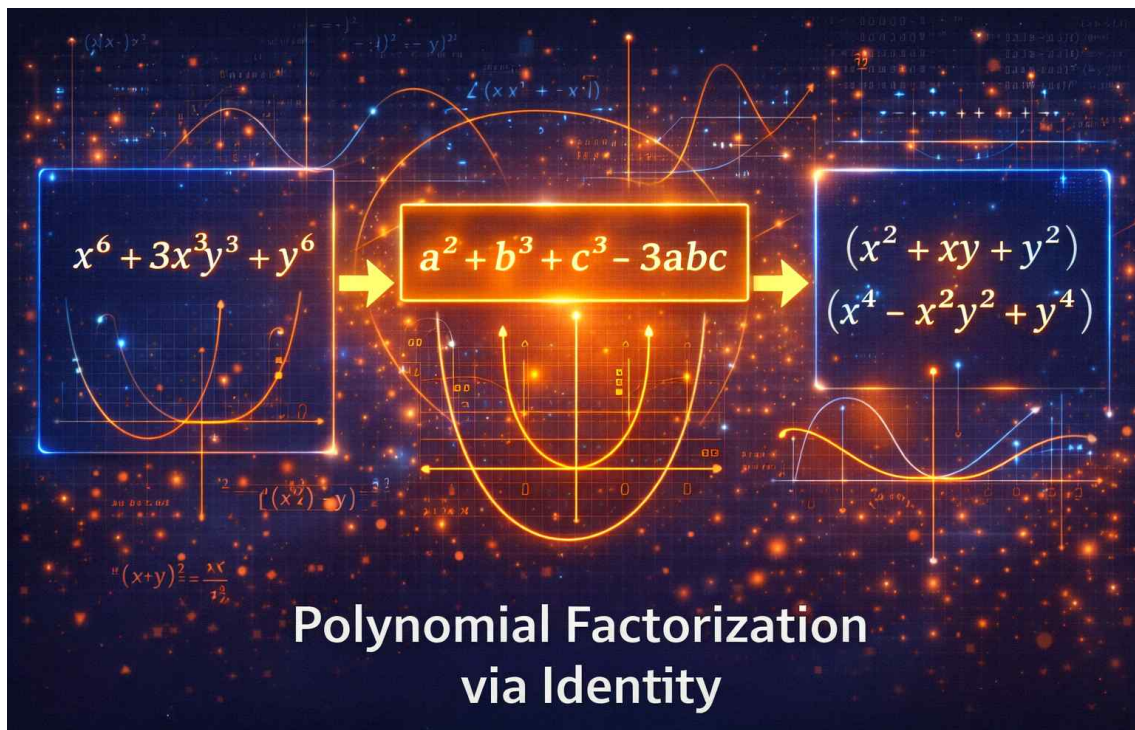
Abstract

This paper explores new ways of factorizing polynomials that appear at first to be unfactorable. Beginning with the classical identity

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

we construct novel factorizations by substituting suitable expressions for x, y, z , that cause certain terms to cancel out. We first review known results such as the factorization of $a^4 + 4b^4$, which is obtained by adding and subtracting special terms. We then develop two main approaches: cancellation between two cubic terms, and cancellation between one cubic term and the $3xyz$ term. These yield families of identities depending on a parameter m , and by substituting particular values of m , we generate new explicit factorization formulas. Finally, we analyze the general case using the cubic equation $m^3 - 3m = N$ and show how each real number N corresponds to a specific factorization. This study highlights how classical identities can be extended to create surprising new factorization results.

Keywords: Polynomial factorization, algebraic identities, cubic equations, parameterization, cancellation method



1. Introduction

Factorization is the process of expressing a polynomial as a product of simpler factors, and it plays a very important role in mathematics. Factorization is not only a computational technique but also a method directly connected to finding the solutions of equations. For example, when solving $x^2 - 5x + 6 = 0$, we can factor it as $(x - 2)(x - 3) = 0$, which immediately gives the solutions $x = 2$ and $x = 3$. Therefore, factorization can be considered a central tool in problem solving.

However, some polynomials appear to be impossible to factorize at first glance. For instance, $a^4 + 4b^4$ seems unfactorable, but by adding and subtracting $4a^2b^2$, we can use the difference of squares formula $x^2 - y^2 = (x - y)(x + y)$ to factorize it. In this way, certain ideas and techniques make it possible to factorize expressions that initially seem impossible.

In this study, we extend this perspective by considering the following interesting identity involving three variables:

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

By applying this factorization formula, we aim to construct new examples of polynomials that can be factorized, showing how seemingly complicated or unfactorable expressions can, in fact, be reduced to simpler structures. Through this, we emphasize the importance of factorization not only as a manipulation technique but also as a fundamental process for finding solutions.

2. Known Results

As mentioned in the Introduction, one example of an expression that seems unfactorable at first but can actually be factorized is $a^4 + 4b^4$. Since it is a sum of two positive terms, it appears to resist factorization. However, by adding and subtracting $4a^2b^2$, the expression can be transformed as follows:

$$a^4 + 4a^2b^2 + 4b^4 - 4a^2b^2 = (a^2 + 2b^2)^2 - (2ab)^2 = (a^2 + 2b^2 + 2ab)(a^2 + 2b^2 - 2ab).$$

This demonstrates that by cleverly adding and subtracting certain terms, we can rewrite an expression into a form that is already known to be factorable.

In this case, we applied the identity

$$x^2 - y^2 = (x - y)(x + y),$$

where $x = a^2 + 2b^2$ and $y = 2ab$. The reason why $a^4 + 4b^4$ emerges from this substitution is that the $4a^2b^2$ term from expanding x^2 and the $4a^2b^2$ term from expanding y^2 cancel each other out.

When such cancellation occurs, it becomes difficult to recognize the original form of the factorization. Nevertheless, if we deliberately substitute carefully chosen terms into a known identity so that some terms cancel out, we may create new multiplication formulas that look unfamiliar but are, in fact, derived from classical factorization identities.

3. Construction of New Formulas

In this section, we will use the factorization formula of $x^3 + y^3 + z^3 - 3xyz$ to construct new factorization identities. Recall that the expression is factorized as follows:

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

To apply this formula, we plan to substitute suitable expressions for x , y , and z . As in the example given in Chapter 2, the goal is to create cancellations among certain terms. Specifically, we want terms from x^3 , y^3 , z^3 , and $3xyz$ to cancel with each other.

There are two main ways to achieve such cancellations. The first method is to make two of the cubic terms cancel out. The second method is to arrange for one cubic term to cancel with the $3xyz$ term. In the following, we will investigate what kinds of new factorization formulas can be obtained in each of these cases.

3.1. The Case Where z^3 and x^3 Cancel

To make the terms x^3 and z^3 cancel partially, it is enough to substitute $z = kx$, where k is a constant. Substituting this into $x^3 + y^3 + z^3 - 3xyz$, we obtain

$$(k^3 + 1)x^3 + y^3 - 3kx^2y.$$

On the other hand, substituting $z = kx$ into the right-hand side

$(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$ gives

$$((k + 1)x + y)((k^2 - k + 1)x^2 - (k + 1)xy + y^2).$$

Therefore, we obtain the following factorization formula

$$(k^3 + 1)x^3 + y^3 - 3kx^2y = ((k + 1)x + y)((k^2 - k + 1)x^2 - (k + 1)xy + y^2).$$

In fact, this formula can also be proved directly without using $x^3 + y^3 + z^3 - 3xyz$. We can rewrite $(k^3 + 1)x^3 + y^3 - 3kx^2y$ by factoring out y^3 as follows

$$y^3((k^3 + 1)t^3 - 3kt^2 + 1),$$

where $t = \frac{x}{y}$. Here, $(k^3 + 1)t^3 - 3kt^2 + 1$ is simply a cubic polynomial in t , which can be factorized more directly. In this sense, the formula above is not truly a new identity but rather an alternative proof of a factorization that can already be established. However, as we will see, the next case produces a more interesting result.

3.2. The Case Where z^3 and $3xyz$ Cancel

In this situation, we consider the case where z^3 and $3xyz$ partially cancel each other. To achieve this, we assume that

$$z^3 = kxyz,$$

where k is a constant. Dividing both sides by z , we obtain

$$z^2 = kxy.$$

This condition will serve as the basis for constructing new factorization identities in this case.

In order to satisfy $z^2 = kxy$, it must hold that $z = \pm \sqrt{kxy}$. To simplify the form of z , we can substitute $x = a^2$, $y = b^2$, and $\pm \sqrt{k} = m$. Here m can be positive or negative, so it can be any real number. Then we obtain $z = mab$.

Therefore, let us substitute $x = a^2$, $y = b^2$, and $z = mab$ into the original factorization formula. Then we obtain the following expression:

$$a^6 + b^6 + (mab)^3 - 3ma^3b^3 = (a^2 + b^2 + mab)(a^4 + b^4 + (mab)^2 - a^2b^2 - mab^3 - ma^3b).$$

We can simplify both sides as follows:

$$a^6 + (m^3 - 3m)a^3b^3 + b^6 = (a^2 + mab + b^2)(a^4 - ma^3b + (m^2 - 1)a^2b^2 - mab^2 + b^4).$$

Now, in the next chapter, we will substitute various values of m to create new factorization formulas.

4. New Factorization Formulas from Parameter Choices

In this section, we will consider several values of m into the following formula:

$$a^6 + (m^3 - 3m)a^3b^3 + b^6 = (a^2 + mab + b^2)(a^4 - ma^3b + (m^2 - 1)a^2b^2 - mab^2 + b^4).$$

4.1. $m = 1, m = -1$ case

For this case, we have

$$a^6 - 2a^3b^3 + b^6 = (a^2 + ab + b^2)(a^4 - a^3b - ab^2 + b^4)$$

and

$$a^6 + 2a^3b^3 + b^6 = (a^2 - ab + b^2)(a^4 + a^3b + ab^2 + b^4).$$

Actually, the left hand side of them can be simplified into $(a^3 \mp b^3)^2$, respectively, this formula is not that useful.

4.2. $m = 2, m = -2$ case

For this case, the left hand sides of them are again $a^6 + 2a^3b^3 + b^6$ and $a^6 - 2a^3b^3 + b^6$, respectively. Therefore, the obtained formula is not that useful again.

4.3. $m = 3$

For this case, we have

$$a^6 + 18a^3b^3 + b^6 = (a^2 + 3ab + b^2)(a^4 - 3a^3b + 8a^2b^2 - 3ab^2 + b^4).$$

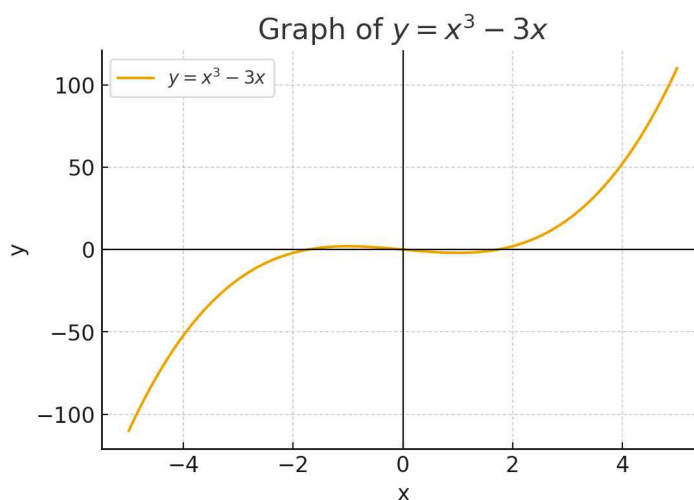
Somehow, this formula might be useful since finding a complicated factor $a^2 + 3ab + b^2$ cannot be easy to guess.

5. General case

Now, we will finalize this paper by considering the general case of

$$a^6 + (m^3 - 3m)a^3b^3 + b^6 = (a^2 + mab + b^2)(a^4 - ma^3b + (m^2 - 1)a^2b^2 - mab^2 + b^4).$$

Let $a^6 + Na^3b^3 + b^6$ be given for some real number N . Recall the graph of $y = x^3 - 3x$ can be drawn as follows:



As we can see in the graph, the range of the graph is the set of all real numbers. Therefore, we know there exists at least one real number m satisfying $N = m^3 - 3m$. Also, due to $m^3 - 3m - N = 0$ is a cubic equation, there is a way to find one solution m .

Then, we can substitute such m into

$$a^6 + Na^3b^3 + b^6 = (a^2 + mab + b^2)(a^4 - ma^3b + (m^2 - 1)a^2b^2 - mab^2 + b^4)$$

to factorize $a^6 + Na^3b^3 + b^6$ into two parts.

For instance, if $N = 4$, we have

$$m = (2 + 3^{1/2})^{1/3} + (2 - 3^{1/2})^{1/3}.$$

Although this is quite complicated, we can factorize $a^6 + 4a^3b^3 + b^6$ into $a^2 + ((2 + 3^{1/2})^{1/3} + (2 - 3^{1/2})^{1/3})ab + b^2$ and the remaining part.

6. Conclusion

In this paper, we have investigated how certain polynomials that initially appear unfactorable can, in fact, be factorized through the careful use of classical identities. Starting from the example of a^4+4b^4 , we showed how the addition and subtraction of terms make hidden structures visible. Using the identity $x^3+y^3+z^3-3xyz$, we constructed new factorizations by introducing cancellations among selected terms. Two distinct methods are used: (1) cancellation between cubic terms and (2) cancellation between one cubic term and $3xyz$. By substituting particular values of the parameter, we obtained several explicit factorizations, some of which reduced to familiar forms, while others produced less obvious but interesting results. Finally, we generalized the approach using the cubic equation $m^3-3m=N$, establishing a link between each real parameter N and a specific factorization. This demonstrates how algebraic creativity, combined with classical identities, can continue to generate new and insightful mathematical results.

7. References

- [1] Hall, H. S., & Knight, S. R. (2007). Higher Algebra. London: Macmillan.
- [2] Stewart, I. (2015). Galois Theory (4th ed.). Chapman & Hall/CRC.
- [3] Stillwell, J. (2010). Mathematics and Its History (3rd ed.). Springer.

Axiom Journal

Can Mathematical Metrics Detect Bias in AI Decisions?

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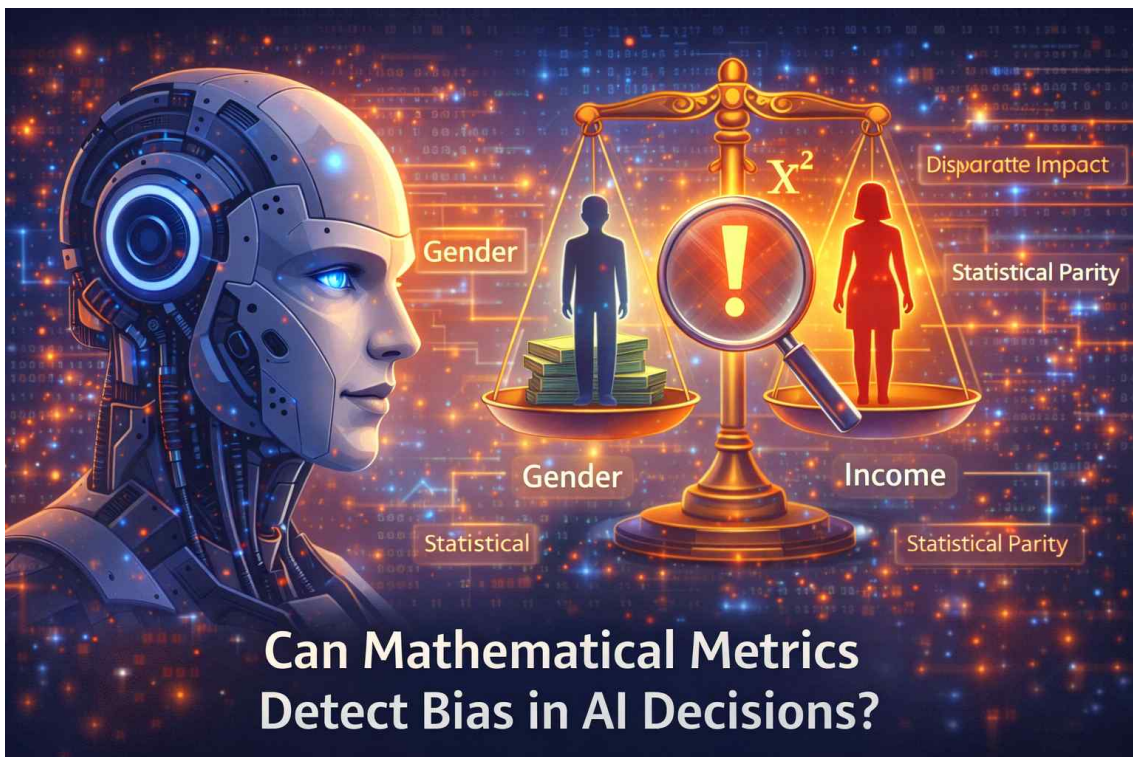


Can Mathematical Metrics Detect Bias in AI Decisions?

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Abstract

This study assesses the accuracy of mathematical metrics for detecting AI decision-making biases across datasets. With the emerging presence of AI in the modern world, the detection of AI misinformation has become more critical than ever, with AI bias being a central issue. This study analyzes the effectiveness of mathematical metrics for detecting gender bias in AI using a dataset from the UC Irvine repository, employing several methods to test for biases in gender and income data. The mathematical metrics used to detect the AI bias were the Chi-Square test, disparate impact, and statistical parity. The Chi-Square test is the primary method used to detect bias in the AI's decision-making. The analysis found that the AI has a strong bias toward men in the UC Irvine dataset. The AI tended to show a bias towards the gender with the higher count in the dataset, revealing an inherent flaw in using AI to make decisions.



Introduction

With Artificial Intelligence embedded in our daily lives, influencing education, decision-making, and media, it is crucial to distinguish between factual information and misinformation. One of the main issues of Artificial Intelligence is that it uses data from the entire internet. Not all sources on the internet are trustworthy, yet AI sometimes uses that information and displays it to the user as factual and correct. This typically happens when AI systems learn from data created by humans, which can lead them to summarize information as an inequality or stereotype based on the data they find. When this biased information is used in real-world applications, such as hiring, loan approvals, or risk assessments, it can result in unfair decisions and controversial outcomes.

Mathematical analysis is a tool that can help detect bias in AI datasets and overall information, allowing us to measure differences between groups using clear, objective numbers. These tools, such as fairness metrics and statistical tests, make it possible to identify whether one group is receiving different outcomes than another, and often whether those differences are larger than chance would predict. Datasets can often be misinterpreted as reflecting stereotypes and inequality if not understood correctly. For example, if a dataset shows that far fewer women earn a given level of high income than men do, a trained AI on that data may mistakenly learn that men are more qualified than women. When patterns like this aren't carefully analyzed, AI systems might take that inequality into account when making decisions, leading to entirely stereotypical choices driven by the dataset's biases.

The overall goal of this project is to use mathematical methods to examine whether gender bias exists in a commonly used income dataset that AI models could potentially be trained on. Ultimately answering the question, “Can Mathematical Methods detect bias within an AI dataset?”

Methods

In this research, we investigate mathematical bias detection in AI decision-making by analyzing whether gender influences income predictions within public datasets. The dataset we used was taken from the UCI Machine Learning Repository's Adult Income Dataset, a source often used to train and test artificial intelligence and run other experiments. This dataset contains real demographics and income information collected in the United States from the 1994 U.S. Census Bureau Database.

In this project, the primary focus was narrowed to two variables within the dataset: gender and income category. The gender variable has only two groups, male and female, while the income variable classifies individuals into two categories: earnings of more than \$50,000 per year or \$50,000 or less per year. These two variables were chosen because they enable clear, measurable analysis of potential gender-related biases in a dataset often used for AI training. Not only that, these two biases are commonly stereotyped in the real world, which would be great for this experiment.

Information Gathered from the Dataset:

Gender and Earnings	Amount of people
Men Earning > 50k	15,127
Men Earning \leq 50k	6,662
Women Earning > 50k	1,179
Women Earning \leq 50k	9,592

Several mathematical metrics were applied to detect bias in AI decision-making. Demographic parity was used to analyze the probability that either men or women would earn more than \$50k a year. We defined the protected attribute as each group's gender and identified the positive outcome as earning \$50k or more per year. The next step was to determine whether men and women had an equal chance of earning more than \$50k per year. This was done by simply calculating the percentage of men who earned more than \$50k per year and comparing it to the percentage of women who did. Statistical parity was then used to assess the difference in high-income rates between men and women. This essentially means we subtracted the percentage of men who earned more than \$50k per year from the percentage of women who earned more than \$50k per year to determine the difference.

The following method utilized was determining the disparate impact ratio. We simply divided the percentage of women who earned \$50k or more by the percentage of men who earned \$50k or more to get the ratio. The final method to be employed was the Chi-Square Test of

Independence. We first created a contingency table from the dataset. We then calculated the expected counts, determining how many of each gender would make above \$50k per year if gender and income were unrelated. The next step was to calculate the four parts of the Chi-Square statistic, then add them to determine whether the difference between gender and income occurs only by chance.

Results

The analysis and methods we used evaluated whether income outcomes differed between men and women using the following metrics: Demographic Parity, Statistical Parity Difference, Disparate Impact, and the Chi-Square Test of Independence (the two-square method). The counts extracted from the Adult Income Dataset from UC Irvine's Machine Learning Repository were used to calculate probabilities, fairness measurements, and the Chi-square Statistic.

DISCLAIMER: Each decimal is rounded to the nearest thousandths

1. Demographic Parity (Probability of Earnings > \$50k)

To first determine whether men and women had an equal chance of earning more than \$50,000 was to examine the proportion of each gender in the high-income category, which was calculated below:

- Men:

$$P(\text{male} > 50k) = \frac{15127}{15127 + 6662} = 0.694$$

- Women:

$$P(\text{female} > 50k) = \frac{1179}{1179 + 9592} = \text{results}$$

These results suggest that men were significantly more likely than women to earn over \$50k.

2. Statistical Parity Difference

This Mathematical Metric measures the difference between the high-income rates of men and women:

$$\text{Statistical Parity Difference} = P(\text{male} > 50k) - P(\text{female} > 50k) = 0.694 - 0.109 = 0.585$$

A difference of 0.585 suggests a significant disparity between the two.

3. Disparate Impact

This Mathematical Metric will help compare the likelihood of women earning > 50k to that of men:

$$\text{Disparate Impact} = \frac{0.109}{0.694} = 0.157$$

A Disparate Impact value below 0.8 indicates potential bias in the Dataset. At 0.157, this result suggests a substantial imbalance.

4. Chi-Square Test of Independence

To determine whether gender and income were statistically related rather than at random, the

chi-square test was applied to the research for further indication. The Chi-square test is a more complex process than the other mathematical metrics, so it is broken down into five steps.

Step 1: Create a contingency table

Gender	>50k	≤50k
Male	15,127	6,662
Female	1,179	9,592

Step 2: Calculate expected counts

Using the table and calculating the totals for each of the genders, along with the income of each of them. Calculate the expected counts: what the numbers would be if gender and income were unrelated.

Expected value: Male > 50k

$$E = \frac{(21,789) (16,306)}{32,560} = 10,907.899$$

Expected Value: Male ≤ 50k

$$E = \frac{(21,789) (16,254)}{32,560} = 10,877.100$$

Expected Value: Female > 50k

$$E = \frac{(10,771) (10,306)}{32,560} = 5,394.100$$

Expected Value: Female ≤ 50k

$$E = \frac{(10,771) (16,254)}{32,560} = 5,376.899$$

Step 4: Compute each Chi-Square piece

Male > 50k

$$\frac{(15127 - 10907.07)^2}{10907.700} = \frac{4,219.3^2}{10,881} = 1633.4$$

Male ≤ 50k

$$\frac{(6662 - 10881.3)^2}{10,881.3} = \frac{4,219.3^2}{10,907.3} = 1636.8$$

Female > 50k

$$\frac{(1179 - 6398.3)^2}{5398.2} = \frac{4,219.3^2}{5398.3} = 3294.8$$

Female ≤ 50k

$$\frac{(9592 - 5372.7)^2}{5372.7} = \frac{4,219.3^2}{5372.7} = 3312.8$$

Step 5: Add the four values together

$$x^2 = 1633.4 + 1636.8 + 3294.8 + 3312.8$$
$$x^2 = 9,877.8$$

Final Chi-Square Results:

Chi-square= 9877.8

Degrees of Freedom = 1

P-value < 0.00001

There is a strong statistical relationship between gender and income level, indicating that the probability this difference occurred by chance is zero.

Discussion

These results were obtained from mathematical metrics calculated from the Adult Income Dataset, indicating a gender bias, meaning the AI system will conclude that stereotypes exist in these topics. Men were found to have earned above \$50k at a much higher rate than women within this dataset. The Statistical Parity Difference shows a 58.5% gap, and the Disparate Impact Ratio of 0.157 is far below the fairness guideline of 0.8, indicating unequal outcomes.

The Chi-square test also confirms a statistically significant association between gender and income. This means the income displayed is highly different and unlikely to be due to random chance. Instead, it is thought to reflect both the real structural inequality captured in the dataset and the inequalities an AI model would learn if trained on this data. In real-world systems such as job screening tools or economic prediction models, this type of imbalance could influence decision-making, ultimately leading to the conclusion that men are more efficient workers than women because they are higher earners. Further research on race could be conducted if desired, but it will likely reach a similar conclusion.

Conclusion

The purpose of this research was simply to detect AI biases in decision-making through mathematical metrics. These mathematical metrics indicate that the difference in the percentage of high-earning individuals between the groups is both very large and unlikely to be due to chance (as noted in the disparate impact ratio and the Chi-Square Test of Independence). Every mathematical metric used showed detectable biases in AI decision-making. This remains important because the use of AI to make decisions is becoming increasingly popular. If we can put mathematical metrics to use and detect biases in AI decision-making, we can prevent users of AI models from misinformation.

References

“Welcome to the UC Irvine Machine Learning Repository.” UCI Machine Learning Repository, UC Irvine, archive.ics.uci.edu/. Accessed 9 Nov. 2025.

Axiom Journal

Mathematical Worthiness of the Powerball Lottery

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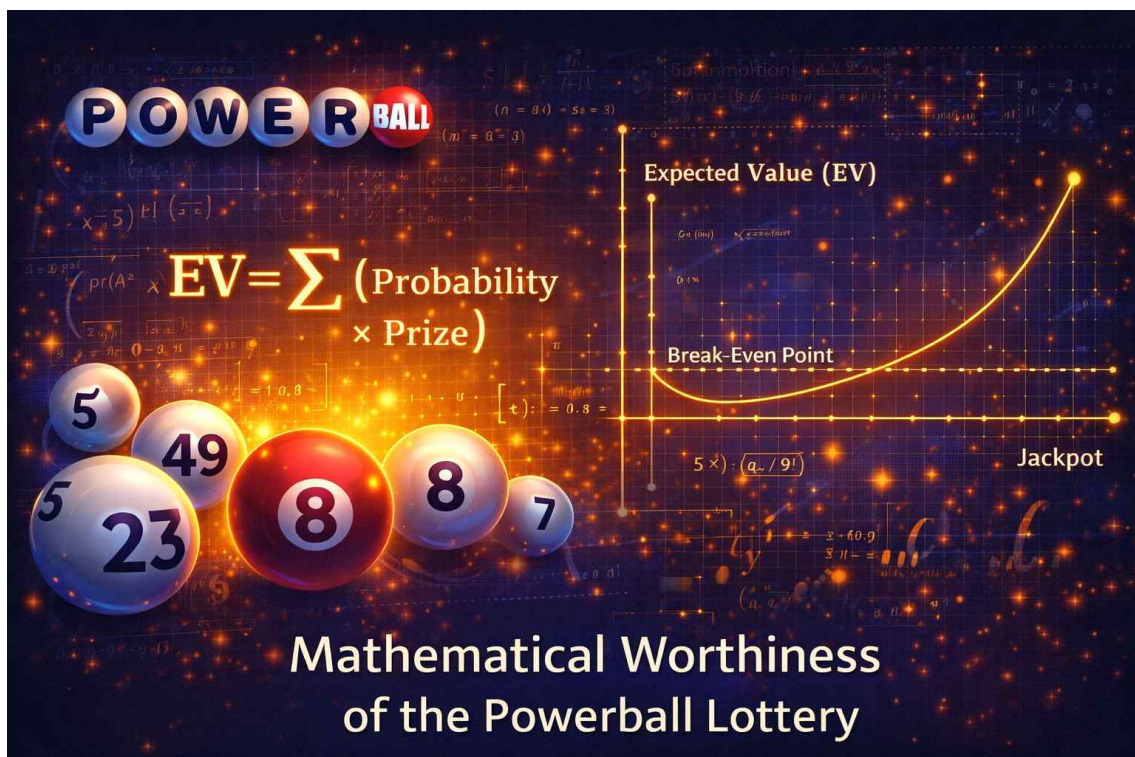
Mathematical Worthiness of the Powerball Lottery

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Abstract

This paper shows that playing Powerball lottery is mathematically worth it. Using probability theory and combinations, we calculate the exact odds of winning each prize and determines the expected value (EV) of different ticket types, including Power Play and Double Play add-ons. Research finds that a basic \$2 ticket becomes “worth it” at jackpots over \$556.7 million, while the \$4 ticket with both add-ons requires nearly \$955.6 million. Similarly, from the real-world case, the \$2.04 billion jackpot in 2022, shows that even when EV is favorable, the odds of winning remain extremely low. The paper concludes that due to taxes, jackpot splitting, and the sheer improbability of winning, lottery tickets should never be used to earn money.

Keywords: Powerball Lottery, Expected Value, Combinatorics & Probability, Mathematical Modeling, Double Play, and Power Play Add-ons



Introduction



Figure 1 Powerball Logo. Photo Credit: DC Lottery

To play the Powerball (Figure 1), the biggest U.S. lottery, a player must select 5 numbers from 1 to 69 (white balls) and one number from 1 to 26 (red Powerball). Each ticket costs \$2, with the option to add “Power Play” for an additional \$1, which multiplies non-jackpot prizes from $2\times$ to $10\times$ depending on jackpot size. In some states, players may also add “Double Play” for another \$1, entering the same numbers into a second drawing with fixed prizes up to \$10 million. In this paper, we calculate the true probabilities of winning each prize using combinations and probability theory. Then, we evaluate the expected value of the basic \$2 ticket, the \$3 Power Play ticket, the \$3 Double Play ticket, and the \$4 ticket with both add-ons to determine the jackpot size at which each ticket becomes mathematically “worth it.” Finally, we apply this model to a real-world case, the \$2.04 billion Powerball jackpot of 2022, and explain why the lottery remains statistically bad to buy even when the expected value appears favorable.

II. Theoretical Background

1. Powerball System



Figure 2 Powerball Game. Photo Credit: ABC News

To play Powerball, the player selects (Figure 2):

- 5 white balls from a pool of 1-69
- 1 red Powerball from a pool of 1-26

The 5 white balls are drawn without replacement, and the Powerball is drawn independently. Each combination is unique, and prize ranks are based on how many numbers match the winning draw.

2. Combination Formula and Its Use

To calculate the probabilities of winning in Powerball, we use the combination formula:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

This formula tells us how many unique ways we can choose r items from a larger set of n items when order does not matter. The exclamation mark (!) denotes a factorial, which means the product of all positive integers up to that number. For example,

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$$

Here are two real-world examples to show how combinations work:

Examples 1: Choosing a Committee

Suppose a school club has 10 members, and they need to form a 3 person per committee. The order of selection does not matter, and being picked first does not automatically make you become a president, so we use combinations:

$$\binom{10}{3} = \frac{10!}{3!(10-3)!} = 120$$

There are 120 different ways to choose 3 people from a group of 10.

Example 2: Drawing 5 Cards from a Deck

A standard deck has 52 cards. If you want to know how many possible 5-card poker hands can be dealt (ignoring order), you calculate:

$$\binom{52}{5} = \frac{52!}{5!(52-5)!} = 2,598,960$$

So, there are 2,598,960 different 5-card combinations in poker.

These kinds of calculations are essential for evaluating odds in games of chance, including lotteries like Powerball. When we calculate the probability of winning different prize ranks, it is essentially to ask: How many favorable combinations meet the win condition? Then, we divide that by the total number of possible combinations to get the exact probability. Understanding how combinations work allows us to model the Powerball system logically and precisely, rather than relying on estimates or assumptions.

3. Prize Ranks and Match Requirements

Table 1 Prize Ranks and Match Requirements

Rank	Match Requirement
1st	5 white + 1 Powerball
2nd	5 white only
3rd	4 white + 1 Powerball
4th	4 white only
5th	3 white + 1 Powerball
6th	3 white only
7th	2 white + 1 Powerball
8th	1 white + 1 Powerball
9th	0 white + 1 Powerball

III. Research Content

1. Calculation of Probability for Each Rank

We calculate the number of combinations for each rank using combinations:

- Total possible combinations:

$$\binom{69}{5} \times 26 = 292,201,338$$

a. 1st Place: 5 white + Powerball

Only 1 exact match exists

$$P = \frac{1}{292,201,338}$$

b. 2nd Place: 5 white only

$$P = \frac{25}{292,201,338}$$

c. 3rd Place: 4 white + Powerball

Choose 4 out of 5 correct white balls:

$$\binom{5}{4} = 5$$

Choose 1 wrong ball from the remaining 64:

$$\binom{64}{1} = 64$$

$$\text{Total Combinations} = 5 \times 64 \times 1 = 320$$

$$P = \frac{320}{292,201,338}$$

d. 4th Place: 4 white only

Same 5×64 combinations as above, but miss Powerball (25 ways):

$$\text{Total Combinations} = 5 \times 64 \times 25 = 8,000$$

$$P = \frac{8,000}{292,201,338}$$

e. 5th Place: 3 white + Powerball

Choose 3 out of 5 correct white balls:

$$\binom{5}{3} = 10$$

Choose 2 wrong balls from the remaining 64:

$$\binom{64}{2} = 2016$$

$$\text{Total Combinations} = 10 \times 2,016 \times 1 = 20,160$$

$$P = \frac{20,160}{292,201,338}$$

f. 6th Place: 3 white only

Same $10 \times 2,016$ combinations as above, but miss Powerball (25 ways):

$$\text{Total Combinations} = 10 \times 2,016 \times 25 = 504,000$$

$$P = \frac{504,000}{292,201,338}$$

g. 7th Place: 2 white + Powerball

Choose 2 out of 5 correct white balls:

$$\binom{5}{2} = 10$$

Choose 3 wrong balls from the remaining 64:

$$\binom{64}{3} = 41,664$$

$$\text{Total Combinations} = 10 \times 41,664 \times 1 = 416,640$$

$$P = \frac{416,640}{292,201,338}$$

h. 8th Place: 1 white + Powerball

Choose 1 out of 5 correct white balls:

$$\binom{5}{1} = 5$$

Choose 4 wrong balls from the remaining 64:

$$\binom{64}{4} = 635,376$$

$$\text{Total Combinations} = 5 \times 635,376 \times 1 = 3,176,880$$

$$P = \frac{3,176,880}{292,201,338}$$

i. 9th Place: 0 white + Powerball

Choose 0 out of 5 correct white balls:

$$\binom{5}{0} = 1$$

Choose 5 wrong balls from the remaining 64:

$$\binom{64}{5} = 7,624,512$$

$$\text{Total Combinations} = 1 \times 7,624,512 \times 1 = 7,624,512$$

$$P = \frac{7,624,512}{292,201,338}$$



2. Expected Value for No Add-on Ticket (\$2)

Let J be the jackpot. The expected value:

$$EV = \frac{J}{292,201,338} + EV_{fixed}$$

where:

$$EV_{fixed} = \frac{25 \times 1,000,000 + 320 \times 50,000 + 8,000 \times 100 + \dots + 7,624,512 \times 4}{292,201,338} \approx 0.0955$$

To find the break-even jackpot:

$$\frac{J}{292,201,338} + 0.0955 \geq 2 \approx 556,725,248$$

So, a \$2 No Add-on ticket becomes worth it when $J \geq \$556.7 \text{ million}$

3. Expected Value for Powerplay Ticket (\$3)

Table 2 Power Play Multipliers (\$3)

Jackpot Size	Multipliers (Odds)
$< \$150M$	$10 \times (1/43), 5 \times (2/43), 4 \times (3/43), 3 \times (13/43), 2 \times (24/43)$
$\geq \$150M$	$5 \times (2/42), 4 \times (3/42), 3 \times (13/42), 2 \times (24/42)$

2nd prize is capped at \$2M regardless of multiplier. Only 3rd-9th prizes get multiplied.

- Expected multiplier for : $< \$150M$

$$E[m] = \frac{119}{43} \approx 2.767$$

- For : $\geq \$150M$

$$E[m] = \frac{109}{42} \approx 2.595$$

Total prize value (excluding 2nd):

2nd prize EV :

$$\frac{25 \times 2,000,000}{292,201,338} \approx 0.171$$

Adjusted EV

- $< \$150M$

$$EV \approx \frac{J}{292,201,338} + 0.171 + \frac{2,889,280 \times 2.767}{292,201,338} \approx \frac{J}{292,201,338} + 0.1983$$

Break even:

$$J \geq 820,455,558$$

● $\geq \$150M$

$$EV \approx \frac{J}{292,201,338} + 0.1966$$

$$J \geq 820,934,319$$

A \$3 Power Play ticket is “worth it” when $J \geq \$820.9 \text{ million}$

4. Expected Value for Double Play + Power Play Ticket (\$4)

Double Play is an optional add-on for an additional \$1 that enters the same set of numbers from the main Powerball ticket into a second drawing. This drawing uses the same format (5 white balls from 1 to 69 and 1 red Powerball from 1 to 26), but the prizes are fixed and lower than the main draw. The top prize in Double Play is \$10 million, and there is no progressive jackpot.

Table 3 Expected Value for Double Play and Power Play Ticket (\$4)

Match Requirement	Prize
5 white + 1 Powerball	\$10,000,000
5 white only	\$500,000
4 white + 1 Powerball	\$50,000
4 white only	\$500
3 white + 1 Powerball	\$500
3 white only	\$20
2 white + 1 Powerball	\$20
1 white + 1 Powerball	\$10
0 white + 1 Powerball	\$7

The total number of combinations is the same as the main draw:

$$\binom{69}{5} \times 26 = 292,201,338$$

Because the drawing structure is identical, the probabilities of each prize tier are the same as previously calculated in section III.1.

Using those probabilities and the Double Play prize amounts, we calculate the expected value:

EV=

$$\frac{1 \times 10,000,000 + 25 \times 500,000 + 320 \times 50,000 + 8,000 \times 500 + 20,160 \times 500 + 504,000 \times 20 + 416,640 \times 20 + 3,176,880 \times 10 + 7,624,512 \times 7}{292,201,338}$$

$$= \frac{156,133,184}{292,201,338} \approx 0.5343$$

Now we combine:

- Powerplay *EV* ($\geq \$150M$): 0.1966
- Double Play *EV*: 0.5343
- Total *EV* (non-jackpot):

$$EV_{fixed} = 0.1966 + 0.5343 = 0.7309$$

Now solve:

$$\frac{J}{292,201,338} + 0.7309 \geq 4 \approx 955,590,988$$

So with Power Play and Double Play, the jackpot must be at least to make the \$4 ticket “worth it.”

5. Expected Value for Double Play Ticket (\$3)

If a ticket is a just a double play, its total expected value is:

$$EV = \frac{J}{292,201,338} + 0.5343$$

We solve for:

$$\frac{J}{292,201,338} + 0.5343 \geq 3 \approx 719,915,034$$

A ticket with Double Play only becomes “worth it” when the main jackpot is at least \$719.9 million.

IV. Conclusion

Through combination analysis and expected value modeling, it turns out that there is a jackpot by purchasing a Powerball ticket becomes mathematically “worth it.” For a standard \$2 ticket, the break-even jackpot value is approximately \$556.7 million. For a \$3 Power Play ticket, this “worth-it” point rises to around \$820.9 million, and for a \$3 Double Play ticket (no Power Play), it is about \$719.9 million. Finally, a ticket that includes both Power Play and Double Play becomes “worth it” only when the main jackpot reaches about \$955.6 million. All these break-even values were calculated with no taxes, no split jackpots, full lump-sum payout, based on the assumption that each combination has an equal chance of being selected and not duplicated (which is unrealistic in many ways).



Figure 3 Powerball Result in Altadena, California.

Photo Credit: Powerball Winner in Altadena, California.

However, mathematical analysis alone does not capture the full story. There is a clear gap between theoretical value and practical reality. A real-world example that shows this gap is the \$2.04 billion Powerball jackpot in November 2022 (Figure 3). At that time, the expected value of a \$2 ticket was more than \$7, far exceeding the “worth-it” jackpot of \$556.7 million. From a purely probabilistic standpoint, this made each ticket “worth it.” And yet, despite tens or even hundreds of millions of tickets sold, nobody won for over 40 consecutive drawings. This reveals a fundamental truth: even when the expected value is favorable, the individual chance of success remains very low.

The problem lies in the scale of Powerball. With over 292 million possible combinations, it is possible for hundreds of millions of tickets to be sold across the U.S. without all combinations being covered. Additionally, many players choose the same “lucky numbers,” which leads to duplicated combinations that increase the chance of splitting the jackpot but not the chance of winning. So even when math says a ticket is “worth it,” the odds are still so tiny that individual players are almost guaranteed to lose.



Figure 4 They win every time. Photo Credit: Reddit

On top of that, the analysis excludes taxes, which significantly reduces the final payout in reality. After accounting for the lump-sum reduction and federal and state taxes, winners often receive only about 31.5% of the advertised jackpot. For example, a couple of years ago, a \$1.28 billion jackpot turned into roughly \$433.7 million in usable money (Figure 4). If these factors had been included in the model, the break-even jackpot thresholds would be far higher, possibly making even the 2022 jackpot not “worth it.” This highlights how the Powerball system is ultimately designed to make money for governments and organizers, not individual players.

While Power Play and Double Play both look like they improve prize outcomes, they are not necessarily increasing the chances of receiving prizes. Power Play increases non-jackpot prizes but caps the 2nd prize at \$2 million, and its added \$1 cost raises the jackpot threshold for profitability. Double Play gives a second shot at fixed prizes up to \$10 million, and while its *EV* is slightly higher than Power Play’s, it still falls short of its \$1 cost unless the jackpot is unusually high. Even combining both options into a \$4 ticket only becomes “worth it” if the jackpot is nearly \$1 billion.

Taken together, these findings show that lotteries are not good investments. They are structured to appear fun, exciting, and potentially rewarding, but the statistical model ensures that most players lose repeatedly, regardless of how large the jackpot becomes. The illusion of accessibility and life-changing fortune lures players in, but the odds remain very unrealistic to win.

To sum up, while this model shows there are theoretical points where certain Powerball tickets become “worth it,” this should not be mistaken for financial advice. Real-world variables, like taxes, duplicate entries, jackpot sharing, and gambler psychology make even billion-dollar jackpots statistically unwise. When one wants to buy lottery tickets, it should be for fun, not for making money, cause they will not win much, in a statistical sense.

References

- [1] ABC News. A look back at some of the emotional wins. ABC News, 13 Jan. 2026, abcnews.go.com/US/security-scenes-big-powerball-drawing/story?id=36246467.
- [2] Blitzer, Robert. *Thinking Mathematically*. Pearson, 2018.
- [3] Garfield, Joan. *The Assessment Challenge in Statistics Education*. IOS Press, 1994.
- [4] Haigh, John. *Taking Chances: Winning with Probability*. Oxford University Press, 2003.
- [5] “Powerball Logo.” *DC Lottery*, District of Columbia Office of Lottery and Gaming, dclottery.com/sites/default/files/2024-01/Powerball_DP%20and%20PP_Color.png.
- [6] Powerball \$2.04 Billion Dollar Winning Ticket Sold In Altadena, California. *MidJersey.News*, Dennis Symons, Jr., 8 Nov. 2022, midjersey.news/2022/11/08/powerball-2-04-billion-dollar-wining-ticket-sold-in-altadena-california/.
- [7] Rosen, Kenneth H. *Discrete Mathematics and Its Applications*. McGraw-Hill Education, 2018.
- [8] They win every time. *Reddit*, 2023, preview.redd.it/they-win-every-time-v0-il1zqfspli8c1.jpeg?auto=webp&s=80ad1f188efb91c27a39cb33316e4b41d9225eb2.
- [9] Triola, Mario F. *Elementary Statistics*. Pearson, 2019.



Axiom Journal

Congruency of Triangles Using Two Corresponding Adjacent Sides and an Excluded Angle

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Congruency of Triangles

Using Two Corresponding Adjacent Sides and an Excluded Angle

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Here I introduce a simple congruency criterion for triangles involving their corresponding adjacent sides and an angle which is not included. If in two triangles, corresponding adjacent sides are equal, and angles opposite to the longer of the two sides are equal, then the two triangles are congruent.

The situation in which two triangles can be formed from two sides and an unincluded angle is called the SSA ambiguous case. However, the ambiguity only occurs when the given angle is opposite the shorter of the two given sides.

The provided criterion explores an alternate situation, ie, when the angle taken is opposite the longer side. This case guarantees that a triangle formed with such dimensions will always be unique. Therefore, if a triangle's two mutually unequal sides are equal to the corresponding sides of another triangle along with the angle which is opposite to the longer of the two sides, a congruence can be established.

This paper provides a solution to the SSA ambiguous case using the idea of the uniqueness of construction.

Below two methods of proof are provided, 1) The Geometric Proof, 2) The Algebraic Proof. The geometry to provide a clear visualisation and the algebra to mathematically prove the given result.



Geometric Proof:

If we consider two triangles, each with sides equal to 3cm, 4cm and 5cm, both those triangles will be congruent by SSS congruency criterion. But if we were to construct a triangle using those dimensions, we would be able to construct only one, since all triangles will be congruent. From this we can infer that two triangles will be congruent if, from the given data, only one possible triangle can be constructed.

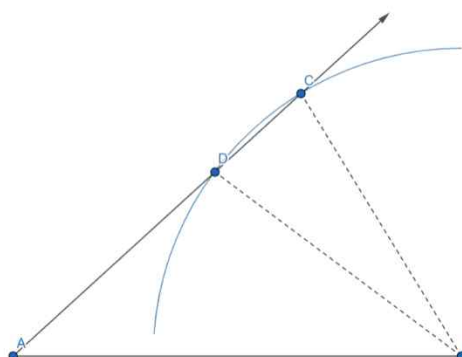
For example, let us construct a triangle ABC with dimensions $AB=16.5\text{cm}$, $BC=11\text{cm}$ and angle $A=40^\circ$

Construction:

Taking B as centre and radius=11cm, we construct an arc. We can see that the arc intersects the ray at two points, C and D. Points C and D are at the same distance from B but at different distances from A, giving two non-congruent triangles ABC and ABD. Thus, two triangles with given dimensions specified are not necessarily congruent.

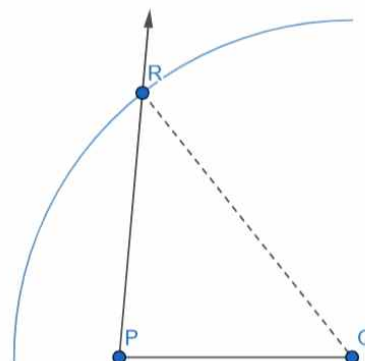
However, the above case occurs only when the given angle lies opposite the shorter of the two given sides.

Let us try by considering dimensions where the common angle is opposite to the longer of the two sides.



Let us construct a triangle PQR with dimensions $PQ=11\text{cm}$, $QR=16.5\text{cm}$ and angle $P=85^\circ$.

Taking Q as centre and radius=16.5cm, we construct an arc. We can see that the arc intersects the ray at only one point P. This proves that for the given dimensions, there is only one possible triangle that can be constructed, therefore, if two triangles share the corresponding dimensions, they will always be congruent.



We can thus see that if the corresponding, unincluded angle is opposite to the longer of the two corresponding, equal sides, the two triangles must be congruent.

We can further try by using concentric circles as the radius of a circle is equal throughout. Let us once again, take the example of triangle PQR.

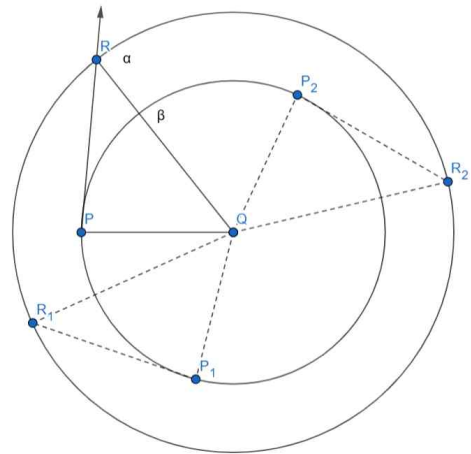
Taking Q as centre, we construct a circle with radius=16.5cm and a smaller circle with radius=11cm.

Let the bigger circle be circle α and the smaller circle be circle β .

We draw a line from centre Q to any point on circle α , say P'. We construct an 85° angle on line QP' meeting circle β at point R'.

On comparing triangles PQR and P'QR', we see that both triangles are identical.

We can attempt to construct as many triangles as possible throughout the arrangement, but the triangle will always be the same.



Algebraic Proof:

Here we use the same approach where two triangles are congruent when only one triangle can be constructed using the given dimensions. However here we prove why it only works if the unincluded angle is taken opposite to the longer of the two corresponding sides.

Let's take a triangle ABC . $a = BC$, $b = AC$, $c = AB$. $\angle C = \theta$

Let us construct an altitude from A , meeting side BC at point D

$$BD = x$$

$$AD = b \sin \theta$$

$$b = \frac{AD}{\sin \theta}$$

Let $BD = x$

$$AD = \sqrt{c^2 - x^2}$$

$$\cos \theta = \frac{DC}{AC}$$

$$DC = AC \cos \theta = b \cos \theta$$

$$x = a - b \cos \theta$$

$$AD = \sqrt{c^2 - a^2 - b^2 \cos^2 \theta + 2ab \cos \theta}$$

$$b = \frac{\sqrt{c^2 - a^2 - b^2 \cos^2 \theta + 2ab \cos \theta}}{\sin \theta}$$

$$b \sin \theta = \sqrt{c^2 - a^2 - b^2 \cos^2 \theta + 2ab \cos \theta}$$

$$b^2 \sin^2 \theta = c^2 - a^2 - b^2 \cos^2 \theta + 2ab \cos \theta$$

$$b^2 (\sin^2 \theta + \cos^2 \theta) = c^2 - a^2 + 2ab \cos \theta$$

$$b^2 - (2a \cos \theta)b - (c^2 - a^2) = 0$$

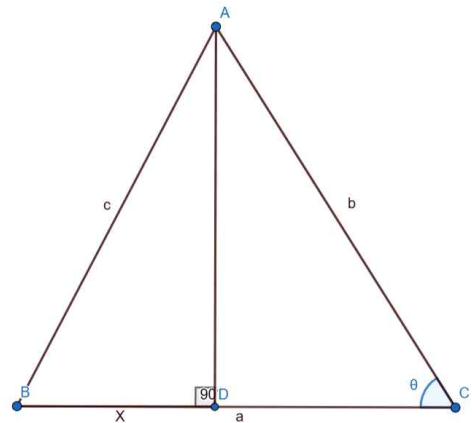
This is a quadratic equation

$$b = \frac{2a \cos \theta \pm \sqrt{4a^2 \cos^2 \theta + 4(c^2 - a^2)}}{2}$$

$$b = a \cos \theta \pm \sqrt{a^2 \cos^2 \theta + (c^2 - a^2)}$$

The \pm represents the two possible values of side b .

The objective is to find a condition such that there is only one possible value for side b .



A value can be eliminated if it is negative, as length cannot be a negative number.

Among the two possible values of b, the one with the minus sign is the only one that can become negative, ie, $a \cos \theta - \sqrt{a^2 \cos^2 \theta + (c^2 - a^2)}$

For this value to be negative, $\sqrt{a^2 \cos^2 \theta + (c^2 - a^2)}$ must be larger than $a \cos \theta$. Thus, the condition is $a \cos \theta < \sqrt{a^2 \cos^2 \theta + (c^2 - a^2)}$.

So we have found the condition which leaves only one possible length of side b. Let us now simplify this.

$$a \cos \theta < \sqrt{a^2 \cos^2 \theta + (c^2 - a^2)}$$

$$a^2 \cos^2 \theta < a^2 \cos^2 \theta + (c^2 - a^2)$$

$$0 < c^2 - a^2$$

$$a^2 < c^2$$

$$a < c \text{ or } -c < -a$$

Thus, if c is greater than a, then there is only one possible value for b.

Thus, we can soundly say that if *in two triangles, two corresponding, adjacent sides are equal and an unincluded angle which is opposite to the longer of the two sides is equal, then the two given triangles are considered congruent.*

Thus, if we have two triangles ABC and PQR where

$$AB = PQ$$

$$BC = QR$$

$$AB > BC \text{ and } PQ > QR$$

$$\angle ACB = \angle PRQ$$

Then triangle ABC and triangle PQR are congruent

This will further be referred to as the *LSA* (Longer side, Shorter side, Angle) criterion, as the given data consists of two adjacent sides of unequal length and an angle taken opposite the longer of the two. The inequality in the lengths of the sides is key, thus it is given importance in the name.

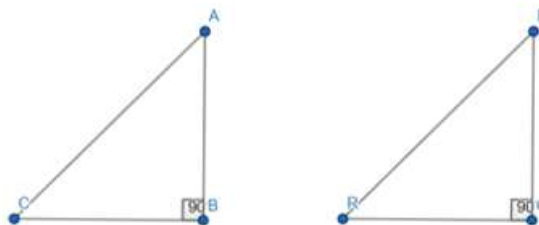
Having established the LSA criterion, it is possible to apply it to certain special cases. In particular, It allows a concise justification of the RHS congruency criterion, which is outlined below.

Lets take a right angled triangle ABC which is right angled at B and another right angled triangle PQR which is right angled at Q It is given that $AB = PQ$ and $AC = PR$. Naturally, $\angle ABC$ is equal to $\angle PQR$ as they are both established to be right angles.

Using LSA, we can state that...

$AC = PR$ (Longer side as Hypotenuse is the longest side in a triangle)

$AB = PQ$ (shorter side)



$\angle ABC = \angle PQR$ (Angle opposite to the longer side)

Thus, triangles ABC and PQR are congruent.

On observing, you can see that RHS can be considered a special instance of LSA, since in every right triangle the hypotenuse is necessarily the longest side, and the right angle is always the angle opposite to it.

Conclusion

We have thus shown that when two triangles share two unequal adjacent sides and the unincluded angle opposite the longer side, the triangles must be congruent. This LSA criterion also naturally includes the RHS case.

References

National Council of Educational Research and Training. (2019). Mathematics - Class IX. NCERT.

National Council of Educational Research and Training. (2020). Mathematics - Class X. NCERT.

Axiom Journal

**Deceptive Odds
: A Mathematical Examination of
How Lotteries Mislead Through Design**

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Deceptive Odds

: A Mathematical Examination of How Lotteries Mislead Through Design

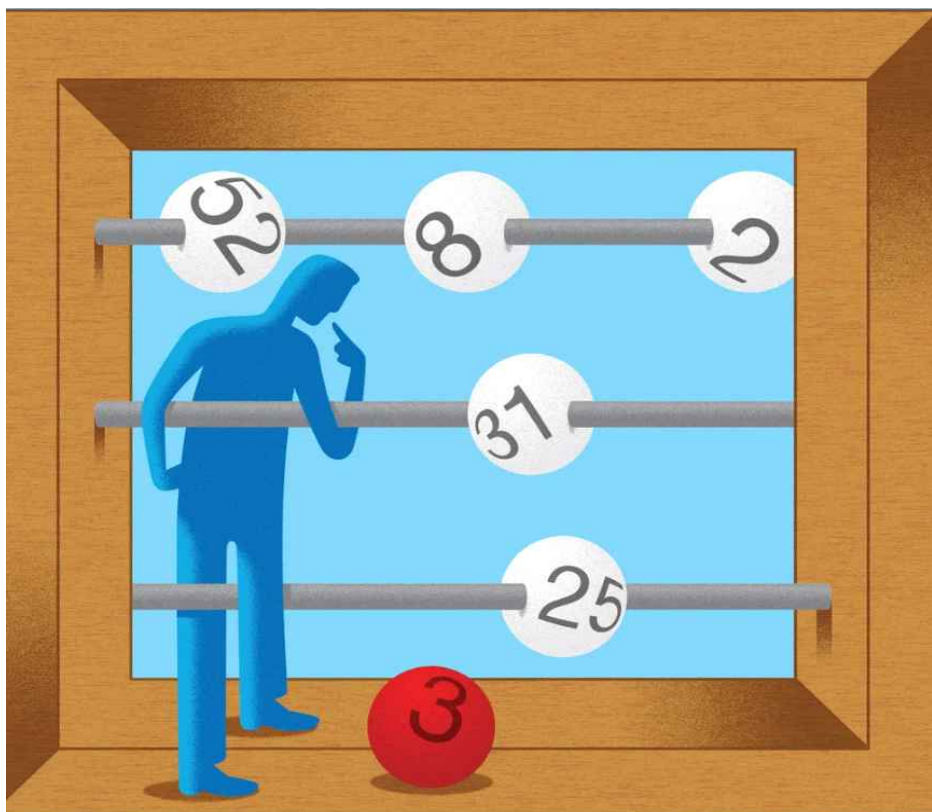
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Abstract

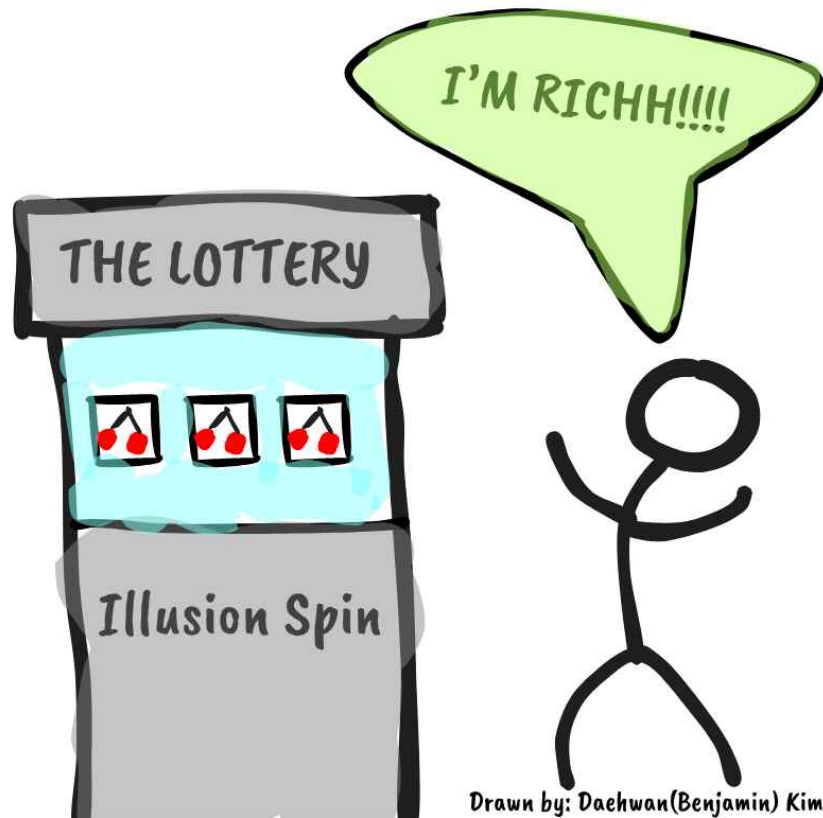
This paper investigates the gap between public perception and mathematical reality in lottery systems by comparing the well known Powerball lottery with a theoretical design called Magic Millions. By calculating the exact probabilities of winning each place, determining the expected value of a \$2 ticket (excluding the jackpot), and computing mathematically fair prize distributions, this study exposes how lottery structures can mislead players. Most people assume that Magic Millions have better odds and fairer but results reveal that while both systems are designed to favor organizers, Powerball offers a relatively more balanced distribution of prizes. In contrast, Magic Millions significantly underpays most prize tiers, often below 2% of their fair value despite presenting a deceptively simple appearance. The findings show how a structure of a lottery can heavily influence perspective on value and fairness.

Keywords: Probability, expected value, lottery fairness, Powerball, Magic Millions, prize structure, perception, illusion.



I. Background

1) Introduction



When most people think about lotteries, they imagine the possibility of winning millions if not billions from a simple \$2 ticket. Especially as we live in an era of social media, it often highlights jackpot winners, creating the illusion that a life changing win is just one lucky ticket away. But what people fail to consider is the mathematical reality behind these games such as how likely is it really to win, are players receiving a fair return for the money they invest, and how can these lotteries become even more lucrative?

I chose this topic because I've wanted to explore the gap between perception and reality, especially in systems designed to attract millions of people. While people realize the chances of a lottery is low, what people fail to realize is how low it really is and that there is a whole another story when it comes to prizes. I want to uncover and understand that imbalance using real numbers, not just assumptions or opinions.

To do this, I decided to compare two lottery systems: the widely-known Powerball, and a theoretical lottery I created called Magic Millions. On the surface, both appear to follow similar mechanics: players select a set of numbers and wait for a random draw to determine winners

across multiple ranks. But beneath that simplicity lies a complex system of probabilities, combinations, and payout distributions that determines how fair or unfair each lottery truly is.

This paper breaks down both lottery systems in three main steps. First, I calculate the exact probability of winning each prize rank, from first to ninth place. This helps establish just how rare each outcome is. Second, I determine the expected value for a \$2 ticket which is essentially calculating how much value a player gets back on average from each lottery, excluding the jackpot. Finally, I use probability theory to calculate the mathematically fair prize money for each tier, revealing whether players are being overpaid or drastically underpaid relative to how difficult it is to win.

2) Theoretical Background

For this paper, we will have to understand some basic background information.

1. The system of Powerball

In Powerball, 5 numbers between 1 to 69 are chosen and are referred to as the general numbers. One number is additionally picked between 1 to 26 called the bonus number. From this information we can have a ranking.

1st place hits 5 general number 1 bonus number
2nd place hits 5 general numbers 0 bonus number
3rd place hits 4 general numbers 1 bonus number
4th place hits 4 general numbers 0 bonus number
5th place hits 3 general numbers 1 bonus number
6th place hits 3 general numbers 0 bonus number
7th place hits 2 general numbers 1 bonus number
8th place hits 1 general numbers 1 bonus number
9th place hits 0 general numbers 1 bonus number

2. Probability

Probability is a way of measuring how likely something is to happen. It's a number between 0 and 1, or sometimes shown as a fraction or percentage. It cannot be negative or more than 1 as there is no event where there is an over 100% chance of happening or a negative percent chance of happening. For instance, if we had a bag of five identical red and blue marbles the chances of us pulling out a blue marble is 5/10 as we have 5 blue marbles out of 10 total marbles. In probability, choosing refers to picking a certain number of items from a larger group without caring about the order. The equation we use is $nCr = \frac{n!}{(n-r)!r!}$ which means out of n we want to choose r . To explain this better we will demonstrate an example. For instance, Bob has five friends and he wants to choose two of them to win a prize. To him it doesn't matter who is chosen first or second they will both receive the same prize, he only cares about which two are picked. Here we have $5C2$ as out of 5, 2 are chosen. This is written as $\frac{5!}{(3!*2!)}$ which equals 10 meaning there are 10 different ways to choose 2 out of 5.

3. Expected Value

Expected value is a way of calculating how much something is worth on average over time even if the result changes each time. It essentially answers the question If I played this game many times, how much would I win (or lose) on average per play? For instance, Bob is playing a game where he flips a coin. If it lands on head he gets \$2 and nothing if it lands tail. The expected value would be $(.5 * 2) + (.5 * 0) = 1$. The .5 is the probability and the 2

and 0 are the possible values Bob can gain. We end up getting 1 which means on average Bob earns \$1 per flip in the long run.

4. Independent event

Independent events are events where the outcome of one does not affect the outcome of the other. In other words, if we had two independent events, even the results of the first event does not tell you anything about the results in the second. For instance, Bob and his friend Joe are playing a game. Both have a box containing five red and blue balls in front of them. They both blindly pick a ball. The probability of Bob picking a red ball is $5/10 = 1/2$ and the same goes for Joe. Let's say Bob did end up picking a red ball. The probability of Joe picking a red ball has not changed and remains $1/2$. From here to find the probability that both Bob and Joe pick red balls, multiply their individual probabilities: $1/2 * 1/2 = 1/4$.

II. Body

1. We will first calculate the probability of each rank for power ball

The total number of cases is to pick 5 numbers from 69 general numbers and 1 number from 26 bonus numbers. It is important to understand that the probability of the two events is independent because the general number draw and bonus draw are separate. Therefore the total case is $69C5 * 26 = 292201338$.

Now using the total number of cases we can find the probability of each rank.

1. For the probability of first place, since there is only one case as one has to match all 5 general numbers and the bonus.

Therefore, the probability of first place is $1/292201338$

2. Now let's calculate the probability of second place. Second place matches all the general numbers but gets the bonus number wrong. The number of cases should be 1 at first with the 5 general numbers matching up. However, since they got the bonus number wrong the case should be $26-1=25$ as they chose any except the one correct number.

Therefore, the probability of second place is $25/292201338$

3. Now let's calculate the probability of third place. Third place got 4 out of 5 general numbers correct and hit the bonus number. This means that out of 69 numbers, choose 4 out of the 5 winning numbers and 1 out of $69-5 = 64$ numbers instead of the winning number. The number of cases for the bonus would be one as they get it correct. Therefore, we get $5C4 * 64C1$ as the total number of cases = 320

Therefore, the probability of third place is $320/292201338$

4. Now let's calculate the probability of 4th place. For fourth place, one must hit 4 out of 5 general numbers while the bonus number is wrong. This is the same as third place except they got the bonus number wrong. Therefore, we can use the previous $5C4 * 64C1 = 320$ for the general numbers. Since they chose a bonus number among 26 numbers except the one correct number, so the number is 25. We end up getting $5C4 * 64C1 * 25 = 8000$.

Therefore, the probability of 4th place is $8000/292201338$

5. Now let's calculate the probability of 5th place. For fifth place, one must hit 3 out of 5 general numbers and hit the bonus number. This means that they chose 3 out of the 5 numbers,

5C3, then chose 2 out of the 69-5=64 wrong number, 64C2. Since they got the bonus number correct, the case number for the bonus event is 1. We end up getting $5C3 * 64C2 * 1 = 20160$.

Therefore, the probability of 5th place is 504000/292201338

6. Now let's calculate the probability of 6th place. For sixth place, one must hit 3 out of 5 general numbers and the bonus number is wrong. As previously calculated, the case for the general number is 20160, but now we will multiply it by 25 as the case for the bonus number is 25 as they chose a number among 26 except a correct 1 number. We end up getting $5C3 * 64C2 * 25 = 504000$.

Therefore, the probability of 6th place is 50400/292201338.

7. Now let's calculate the probability of 7th place. For 7th place, one must hit 2 out of 5 general numbers and hit the bonus number. This means they chose 2 out of the 5 correct numbers, 5C2, and chose 3 out of the 69-5=64 numbers wrong, 64C3. Since they got the bonus right, the case for the bonus would be one. We end up getting $5C2 * 64C3 * 1 = 416640$.

Therefore, the probability of 7th place is 416640/292201338.

8. Now let's calculate the probability for 8th place. For 8th place, one must hit 1 out of the 5 general numbers and hit the bonus number. This means they chose 1 out of the 5 correct numbers, 5C1, and chose 4 out of the 69-5=64 numbers wrong, 64C4. Since they got the bonus right, the case for the bonus would be one. We end up getting $5C1 * 64C4 * 1 = 3176880$.

Therefore, the probability of 8th place is 3176880/292201338.

9. Now let's calculate the probability for 9th place. For 9th place, one got all of the general numbers wrong and hit the bonus number. That means they chose 5 of the 69-5=64 numbers wrong. Since they got the bonus right, the case for the bonus would be one. We end up getting $5C0 * 64C5 * 1 = 7624512$.

Therefore, the probability of 9th place is 7624512/292201338.

We will now do the same but with our theoretical lottery, Magic Millions. The total number of cases is to pick 6 numbers from 30 general numbers and 2 numbers from 10 bonus numbers and another 2 numbers from 10 bonus numbers. It is important to understand that the probability of the two events is independent because the general number draw and bonus draws are separate. Therefore the total case is $30C5 * 10C2 * 10C2 = 1,201,693,125$.

Now using the total number of cases we can find the probability of each rank.

1. For the probability of first place, there is only one case as one has to match all 6 general numbers and all four of the bonus.

Therefore, the probability of 1st place is $1/1,201,693,125$.

2. Now let's calculate the probability of second place. Second place matches all the general numbers but gets the last bonus number wrong. The number of cases should be 1 at first with the 6 general numbers matching up. They also got the first set of bonus meaning that the total case for the first bonus is one. However, since they got one of the second bonus numbers correct, we get $2C1$. They chose one incorrect number out of the $10 - 2 = 8$ numbers, getting $8C1$. We end up getting $2C1 * 8C1 = 16$ as the total number of cases.

Therefore, the probability of 2nd place is $16/1,201,693,125$.

3. Now let's calculate the probability of third place. For 3rd place, one has got all the general numbers and first set of bonuses right. However, they got both of the second bonus numbers wrong. This means out of the $10 - 2 = 8$ wrong numbers they chose 2 leading to $8C2$. We end up getting $8C2 = 28$ as the total number of cases.

Therefore, the probability of 3rd place is $28/1,201,693,125$.

4. Now let's calculate the probability of 4th place. For fourth place, one has got all the general numbers correct. They get one out of two numbers right for the first bonus and get all of the second bonus wrong. There is only one case for the general numbers. For the first bonus set they chose 1 out of the 2 correct number, and 1 out of the $10 - 2 = 8$ wrong numbers leading to $2C1 * 8C1 = 16$. For the second bonus set, they chose 2 out of the $10 - 2 = 8$ wrong numbers leading to $8C2$. We end up getting $1 * 2C1 * 8C1 * 8C2 = 448$.

Therefore, the probability of 4th place is $448/1,201,693,125$.

5. Now let's calculate the probability of 5th place. For 5th place, one has got all the general numbers correct. They got none of the bonus numbers right. There is only one case for the general numbers. For both of the bonus they chose 2 out of the $10 - 2 = 8$ wrong numbers leading to $8C2 * 8C2$. We end up getting $1 * 8C2 * 8C2 = 784$.

Therefore, the probability of 5th place is $784/1,201,693,125$.

6. Now let's calculate the probability of 6th place. For 6th place, one has got 5 out of the 6 general numbers correct. They got all of the bonus numbers right. For the general numbers, they chose 5 out of the 6 numbers and 1 out of the $30 - 6 = 24$ wrong numbers leading to $6C5 * 24C1$. For both of the bonuses we get only one case. We end up getting $6C5 * 24C1 = 144$.

Therefore, the probability of 6th place is $144/1,201,693,125$.

7. Now let's calculate the probability of 7th place. For 7th place, one has got 5 out of the 6 general numbers correct. They got the first set of the bonus correct and one out of two for the second set. For the general, they chose 5 out of the 6 numbers and 1 out of the $30 - 6 = 24$ wrong numbers leading to $6C5 * 24C1$. For the first set of bonuses there is only one case. However, for the second they chose 1 out of the 2 correct numbers and 1 out of the $10 - 2 = 8$ wrong numbers leading to $2C1 * 8C1$. We end up getting $6C5 * 24C1 * 2C1 * 8C1 = 2,304$

Therefore, the probability of 7th place is $2304/1,201,693,125$.

8. Now let's calculate the probability of 8th place. For 8th place, one has got 5 out of the 6 general numbers correct. They got the first set of the bonus correct and missed the second set entirely. For the general, they chose 5 out of the 6 numbers and 1 out of the $30 - 6 = 24$ wrong numbers leading to $6C5 * 24C1$. For the first set of bonuses, there is only one case. However, for the second they chose 2 out of the $10 - 2 = 8$ wrong numbers leading to $8C2$. We end up getting $6C5 * 24C1 * 8C2 = 4,032$.

Therefore, the probability of 8th place is $4032/1,201,693,125$.

9. Now let's calculate the probability of 9th place. For 9th place, one has got 5 out of the 6 general numbers correct. They got the 1 out of the 2 numbers correct for the first set of the bonus and missed the second set entirely. For the general, they chose 5 out of the 6 numbers and 1 out of the $30 - 6 = 24$ wrong numbers leading to $6C5 * 24C1$. For the first set of bonus, they chose 1 out of the 2 correct numbers and 1 out of the $10 - 2 = 8$ wrong numbers leading to $2C1 * 8C1$. For the second they chose 2 out of the $10 - 2 = 8$ wrong numbers leading to $8C2$. We end up getting $6C5 * 24C1 * 2C1 * 8C1 * 8C2 = 64,512$.

Therefore, the probability of 9th place is $64,512/1,201,693,125$.

2. Calculation of fair prize money for Powerball and Magic Millions.

For the powerball, we can calculate the expected value of second to ninth prize excluding the jackpot. The probability of each rank is multiplied by the corresponding prize money, and the sum of them is obtained. Therefore, the expected value of the prize money except jackpot is

$$25/292,201,338 * 1,000,000 + 320/292,201,338 * 50,000 + 8,000/292,201,338 * 100 + 20,160/292,201,338 * 100 + 504,000/292,201,338 * 7 + 416,640/292,201,338 * 7 + 3,176,880/292,201,338 * 4 + 7,624,512/292,201,338 * 4 = 95,626,048/292,201,338 = 0.327.$$

This means that the expected value of the prize money except the jackpot is \$0.327 while the price is \$2 for a single ticket.

For the Magic Millions, we can once again calculate the expected value of second to ninth prize excluding the jackpot. The probability of each rank is multiplied by the corresponding prize money, and the sum of them is obtained. Therefore, the expected value of the prize money except jackpot is

$$16/1,201,693,125 * 1,000,000 + 28/1,201,693,125 * 50,000 + 144/1,201,693,125 * 100 + 448/1,201,693,125 * 100 + 784/1,201,693,125 * 7 + 2304/1,201,693,125 * 7 + 4032/1,201,693,125 * 4 + 64,512/1,201,693,125 * 4 = 17,754,992/1,201,693,125 = 0.0147.$$

This means the expected value of the prize money except the jack pot is \$0.0147 while the price is \$2 for a single ticket.

Examining the two expected values we can see that the expected value for Magic Millions is way lower than the powerball. While expected value helps us understand how much a \$2 ticket is realistically worth, calculating the mathematically fair distribution of prize money allows us to see how underpaid or overpaid each place is. We can then know whether the prize structure aligns proportionally with the difficulty of winning. This will reveal exactly where the lottery favors the organizers and how players may be misled by disproportionate prize structures.

We will assume the expected value of the Magic Millions as \$0.016 for convenience. To distribute the fair amount of prize money mathematically will result in a distribution of \$0.002 for 2nd - 9th place.

Let x_2 be the fair prize money for 2nd place.

$$16/1,201,693,125 * x_2 = 0.002. \text{ Then } x_2 = 150,211.64$$

Let x_3 be the fair prize money for 3rd place.

$28/1,201,693,125 * x3 = 0.002$. Then $x3 = 85,835.22$

Let $x4$ be the fair prize money for 4th place.

$144/1,201,693,125 * x4 = 0.002$. Then $x4 = 16,692.9$

Let $x5$ be the fair prize money for 5th place.

$448/1,201,693,125 * x5 = 0.002$. Then $x5 = 5,364.26$

Let $x6$ be the fair prize money for 6th place.

$784/1,201,693,125 * x6 = 0.002$. Then $x6 = 3,064.02$

Let $x7$ be the fair prize money for 7th place.

$2304/1,201,693,125 * x7 = 0.002$. Then $x7 = 1,043.16$

Let $x8$ be the fair prize money for 8th place.

$4032/1,201,693,125 * x8 = 0.002$. Then $x8 = \$596.06$

Let $x9$ be the fair prize money for 9th place.

$64,512/1,201,693,125 * x9 = 0.002$. Then $x9 = \$37.26$

Now let's compare the original prize money with the fair prize money.

Place	Previous Prize Money	Mathematically calculated fair prize money	Previous prize money/Mathematically calculated fair prize money
2	1,000,000	150,211.64	6.658
3	50,000	85,835.22	0.5827
4	100	16,692.9	0.00599
5	100	5,364.26	0.01865
6	7	3,064.02	0.00229
7	7	1,043.16	0.00671
8	4	\$596.06	0.00671
9	4	\$37.26	0.1073

If we did this exact same thing to powerball we get.

Place	Previous Prize Money	Mathematically calculated fair prize money	Previous prize money/Mathematically calculated fair prize money
2	1,000,000	467522.14	2.14
3	50,000	36525.17	1.37
4	100	1461.01	0.068
5	100	579.76	0.17
6	7	23.19	0.302
7	7	28.05	0.25
8	4	3.68	1.09
9	4	1.53	2.61

These tables reveal a contrast in how each lottery has its prize money relative to the mathematical fairness of each place. In the case of Magic Millions, only the 2nd prize tier exceeds its fair value and by more than 6.6 times. Every other tier is severely underpaid, with most prizes returning less than 1% of what would be considered fair based on their odds. This extreme imbalance showed that the Magic Millions lottery is heavily in favor of the organizers, offering players disproportionately small rewards even for very difficult outcomes. It also reveals that the highest places usually exceed its fair value as most people think about winning the big prizes rather than buying a ticket.

Additionally, Powerball, while still unfair overall, distributes its prize money somewhat more proportionally. The second and third prizes are both overpaid compared to their fair values (by $2.14\times$ and $1.37\times$), and some of the lower-tier prizes (like 8th and 9th) actually pay out slightly more than they statistically should. This suggests that Powerball is designed to make players feel like they have a better chance of winning something “worthwhile,” even if the jackpot remains extremely unlikely.

What these results emphasize is that the structure of a lottery can drastically change its perceived value, even if ticket prices and jackpot sizes seem similar. Magic Millions hides its extreme difficulty behind deceptively smaller numbers for choosing from 30 instead of 69, but as the total number of combinations shows, the sample size is significantly larger, and the chances of winning even mid-tier prizes are worse and the payouts are also less generous.

III. Conclusion

Through this paper, it becomes clear that lotteries are far more misleading and complex than they appear. By breaking down the probability of winning each rank, calculating the expected value of a ticket, and comparing actual prizes with mathematically fair payouts, we were able to uncover how the structure of a lottery affects fairness and perceived value.

The theoretical Magic Millions lottery, although it uses smaller numbers and appears more approachable, actually hides an enormous sample size that drastically reduces the player's chances. Not only are the odds worse, but the rewards for winning especially in lower ranks are heavily underpaid compared to what would be fair based on probability. In fact, most prize tiers in Magic Millions pay out less than 2% of their fair value, with only the 2nd prize dramatically overpaying to create the illusion of generosity. This demonstrates how lottery systems can use psychological design like inflated jackpots and small-looking number pools to attract players while offering extremely low returns.

In comparison, Powerball still heavily favors the organizers but shows a more balanced prize structure. Its second and third prizes are overpaid, and even some lower-tier prizes return values close to or above their mathematically fair amounts. While it remains a losing game on average, Powerball at least distributes rewards in a way that more closely aligns with the difficulty of each rank. It is still clear that the powerball still uses tactics such as the 2nd place prize being overpaid compared to other places to give a false sense of generosity. Ultimately, this paper reveals when examined through a mathematical lens, they expose a system designed to take much more than it gives. So next time you consider buying a lottery ticket, remember that these games are designed to maximize profit for organizers and unless it's a jackpot or second place you are not receiving the fair prize amount. Maybe you should just save your money for investments that offer better value.

References:

[1] Ellenberg, Jordan. *How Not to Be Wrong: The Power of Mathematical Thinking*. Penguin Books, 2014.

[2] “A Good Bet? W&M Professor on the Math & Psychology of Powerball.” *William & Mary News*, 13 Jan. 2016,

[3] Tierney, John. “Is the Lottery Ever a Good Bet?” *Scientific American*, 1 Apr. 2012, <https://www.scientificamerican.com/article/is-the-lottery-ever-a-good-bet/>

[4] Mecklin, Christopher J. “Powerball and the Law of Very Large Numbers.” *Journal of Statistics Education*, vol. 13, no. 2, 2005, <https://jse.amstat.org/v13n2/mecklin.html>

[5] “Expected Value – Definition, Formula, and Solved Examples.” *BYJU’S*,

[6] “Probability – Basic Concepts.” *Khan Academy*, <https://www.khanacademy.org/math/statistics-probability/probability-library>

[7] “Probability of Independent Events.” *Khan Academy*, <https://www.khanacademy.org/math/statistics-probability/probability-library/basic-theoretical-probability/a/probability-of-independent-events>

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